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Course: MATH 104

$$1. \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{11-3x}{x^2+2x-3} dx$$

The denominator = x^2+2x-3

$$= (x^2+3x) - (x-3) = 0$$
$$x(x+3) - (x-3) = 0$$
$$= (x-1)(x+3) = 0$$
$$= \int \frac{11-3x}{(x-1)(x+3)} dx$$
$$\therefore \frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$
$$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$$

Equate the numerator

$$\therefore 11-3x = Ax - A + Bx + 3B$$

Substitute $x = 1$

$$11-3(1) = A(1) - A + B(1) + 3B$$
$$11-3 = A - A + B + 3B$$
$$8 = 4B \quad \therefore B = 2$$

Substitute $x = -3$

$$11-3x = Ax - A + Bx + 3B$$
$$11-3(-3) = A(-3) - A + B(-3) + 3B$$
$$11+9 = -3A - A - 3B + 3B$$
$$\frac{20}{-4} = \frac{-4A}{-4} \quad \therefore A = -5$$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{-5}{(x+3)} + \frac{2}{(x-1)}$$

$$\therefore \int \frac{11-3x}{(x+3)(x-1)} dx = \int \left(\frac{-5}{(x+3)} + \frac{2}{(x-1)} \right) dx$$

$$= \int \frac{-5}{x+3} dx + \int \frac{2}{(x-1)} dx$$

$$= -5 \int \frac{1}{(x+3)} dx + 2 \int \frac{1}{(x-1)} dx$$

$$= -5 \ln|x+3| + 2 \ln|x-1|$$

2 $\int \frac{4x-16}{x^2-2x-3} dx$

The denominator = x^2-2x-3

$$(x^2-3x)+(x-3)=0$$

$$x(x-3)+(x-3)=0$$

$$(x-3)(x+1)=0$$

$$= \int \frac{4x-16}{(x-3)(x+1)} dx$$

$$\therefore \frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

Equate the numerator

$$4x-16 = Ax + A + Bx - 3B$$

Substitute $x = 3$

$$4(3)-16 = A(3)+A+B(3)-3B$$

$$12-16 = 3A+A+0-3B$$

$$-4 = 4A$$

$$\frac{-4}{4} = A$$

$$\therefore A = -1$$

Substitute $x = -1$

$$4x - 16 = A(x+1) + B(x-3)$$

$$4(-1) - 16 = A(-1+1) + B(-1-3)$$

$$-20 = 0 - 4B$$

$$\frac{-20}{-4} = \frac{-4B}{-4} \quad \therefore B = 5$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{-1}{x-3} + \frac{5}{x+1}$$

$$\int \frac{4x-16}{(x-3)(x+1)} dx = \int \left(\frac{-1}{x-3} + \frac{5}{x+1} \right) dx$$

$$= \int \frac{-1}{x-3} dx + \int \frac{5}{x+1} dx$$

$$= -1 \int \frac{1}{x-3} dx + 5 \int \frac{1}{x+1} dx$$

$$= -1 \ln(x-3) + 5 \ln(x+1)$$

$$3 \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

Resolving into partial fractions

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Equate the numerators:

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

Substitute $x = 2$

$$2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$$

$$-45 = 15B$$

$$\frac{-45}{15} = \frac{15B}{15}$$

$$\therefore B = -3$$

Substitute $x = -1$
 $2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$
 $\rightarrow 24 = 6A$
 $\frac{24}{6} = \frac{6A}{6}$
 $\therefore A = -4$

Substitute $x = -3$
 $2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$
 $\frac{10}{-10} = \frac{10C}{-10}$
 $\therefore C = 1$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{-4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \left(\frac{-4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3} \right) dx$$

$$= \int \frac{-4}{x+1} dx - \int \frac{3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= -4 \int \frac{1}{x+1} dx - 3 \int \frac{1}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= -4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3|$$

