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MATRIC N0: 19/ENG05/016
DEPARTMENT: MECHATRONICS ENGINEERING.
COURSE CODE: MAT 102

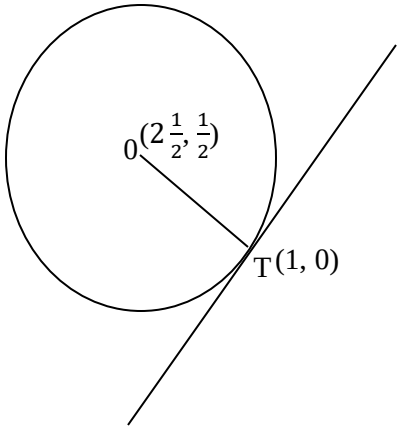
COVID-19 HOLIDAY ASSIGNMENT

1. Find the equation of the tangent at the point (1, 0) on the circle $x^2 + y^2 - 5x - y + 4 = 0$

SOLUTION
Equation of circle $x^2 + y^2 - 5x - y + 4 = 0$

Using completing the square method
$x^2 - 5x + y^2 - y = -4$
$(x - 2.5)^2 + (y - 0.5)^2 = -4 + 6.25 + 0.25$
$(x - 2.5)^2 + (y - 0.5)^2 = -4 + 6.5$
$(x - 2.5)^2 + (y - 0.5)^2 = 2.5$

The centre of the circle is $(2\frac{1}{2}, \frac{1}{2})$



Slope of the Radius of the circle, R = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{0 - \frac{1}{2}}{1 - 2\frac{1}{2}}$$
$$= \frac{-\frac{1}{2}}{-1\frac{1}{2}}$$
$$= \frac{-\frac{1}{2}}{-\frac{3}{2}}$$
$$= \frac{1}{2} \times \frac{2}{3}$$
$$= \frac{1}{3}$$

Slope of the Tangent to the circle, T = $-\frac{1}{m}$

$$= -\frac{1}{\frac{1}{3}}$$
$$= -\frac{1}{1} \times \frac{3}{1}$$
$$= -3$$

The equation of the Tangent: $(y - y_1) = m(x - x_1)$

$$(y - 0) = -3(x - 1)$$
$$y = -3x + 3$$
$$y + 3x - 3 = 0$$

∴ The equation of the Tangent is $3x + y - 3 = 0$

2. Find the equation of the tangent at the point (1, 0) on the circle $x^2 + y^2 - 12x - 12y + 47 = 0$

SOLUTION

Equation of circle $x^2 + y^2 - 12x - 12y + 47 = 0$

Using completing the square method

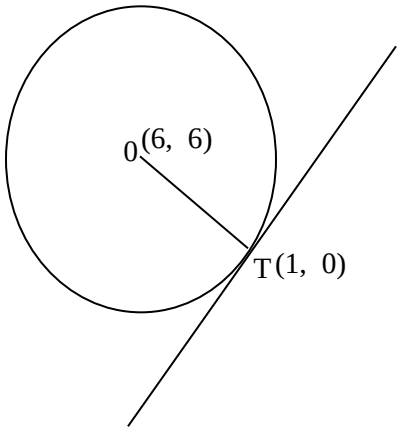
$x^2 - 12x + y^2 - 12y = -47$

$(x - 6)^2 + (y - 6)^2 = -47 + 36 + 36$

$(x - 6)^2 + (y - 6)^2 = -47 + 72$

$(x - 6)^2 + (y - 6)^2 = 25$

The centre of the circle is $(6, 6)$



Slope of the Radius of the circle, R = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{0 - 6}{1 - 6}$$
$$= \frac{-6}{-5}$$
$$= \frac{6}{5}$$

Slope of the Tangent to the circle, T = $-\frac{1}{m}$

$$= -\frac{1}{\frac{6}{5}}$$
$$= -\frac{1}{1} \times \frac{5}{6}$$
$$= -\frac{5}{6}$$

The equation of the Tangent: $(y - y_1) = m(x - x_1)$

$$(y - 0) = -\frac{5}{6}(x - 1)$$
$$y = -\frac{5}{6}(x - 1)$$
$$6y = -5(x - 1)$$
$$6y = -5x + 5$$
$$6y + 5x - 5 = 0$$

∴ The equation of the Tangent is $5x + 6y - 5 = 0$

3. Find the equation of the tangent at the point (1, 0) on the circle $x^2 + y^2 - 8x + 14y + 40 = 0$

SOLUTION

Equation of circle $x^2 + y^2 - 8x + 14y + 40 = 0$

Using completing the square method

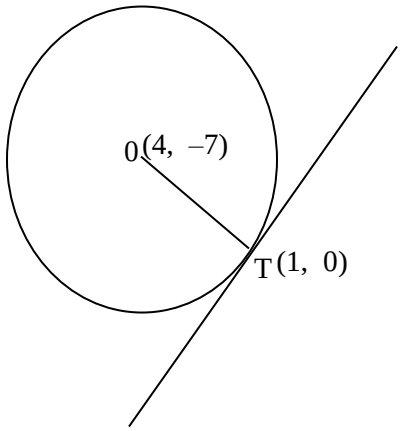
$x^2 - 8x + y^2 + 14y = -40$

$(x-4)^2 + (y+7)^2 = -40 + 16 + 49$

$(x-4)^2 + (y+7)^2 = -40 + 65$

$(x-4)^2 + (y+7)^2 = 25$

The centre of the circle is $(4, -7)$



Slope of the Radius of the circle, R $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0+7}{1-4}$

$= \frac{7}{-3}$

$= -\frac{7}{3}$

Slope of the Tangent to the circle, T $= -\frac{1}{m}$

$= -\frac{1}{-\frac{7}{3}}$

$= -\frac{1}{1} \times -\frac{3}{7}$

$= \frac{3}{7}$

The equation of the Tangent: $(y - y_1) = m(x - x_1)$

$(y - 0) = \frac{3}{7}(x - 1)$

$y = \frac{3}{7}(x - 1)$

$7y = 3(x - 1)$

$7y = 3x - 3$

$7y - 3x + 3 = 0$

∴ The equation of the Tangent is $-3x + 7y + 3 = 0$