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DEPT : CIVIL ENGINEERING

MAT. NO. : 18/ENG03/055

COURSE : ENGINEERING MATHEMATICS

DATE : 05 - 05 - 2020

ASSIGNMENT.

Question 1.

A.) 1200 gallons of water, 150 lb of salt

amount of salt at any period "t" = m

Using the balance law,

Accumulation rate of salt in tank = Input rate of salt - Output rate of salt.

$m_{in} = 50\,gal/min \times (1 + \sin t)\,lb/gal$

$m_{in} = 50 + 50\sin t\,lb/min$

$m_{out} = 30/1200 \cdot m \cdot lb/min = 0.025m\,lb/min$

$$* \frac{dm}{dt} = (50 + 50\sin t) - 0.025m *$$

(B) $$\frac{dm}{dt} = (50 + 50\sin t) - 0.025m$$

$$\frac{dm}{dt} + 0.025m = 50 + 50\sin t.$$

Using integrating factor;

$y \cdot IF = \int Q \cdot IF \cdot dx.$

$IF = e^{\int P dx}$, $P = 0.025$, $Q = 50 + 50\sin t.$

$y = m.$

$m \cdot e^{\int 0.025 dt} = \int (50 + 50\sin t)(e^{\int 0.025 dt})\,dt$

$m \cdot e^{0.025t} = \int (50 + 50\sin t)(e^{0.025t}) \cdot dt$

$\int (50 + 50\sin t)(e^{0.025t})\,dt$ —— **

~~$50 + 50\sin t\,(40e^{0.025t})$~~

Using integration by parts

$\int u\,dv = uv - \int v\,du$

$= 50 + 50\sin t\,(40e^{0.025t}) - \int 50\cos t\,(40e^{0.025})$

$= 2000e^{0.025t} + 2000e^{0.025t}\sin t - \int 2000\cos t\,e^{0.025t}$ —— ***

Solving "$\int 2000\cos t\,e^{0.025t}$" using by parts.

$2000\int \cos t\,e^{0.025t}$

$= 40\cos t\,e^{0.025t} + 40\int \sin t\,e^{0.025t}$ —— ****

Solving "$40\int \sin t\,e^{0.025t}$" using by parts

$40\int \sin t\,e^{0.025t}$

$= 40\sin t\,e^{0.025t} - 40\int \cos t\,e^{0.025t}$ [substitute $40\int \sin t\,e^{0.025t}$ into eq. ****]

Equate "$\int \cos t\,e^{0.025t}$" to equation (****)

$\int \cos t\,e^{0.025t} = 40\cos t\,e^{0.025t} + 40\left[40\sin t\,e^{0.025t} - 40\int \cos t\,e^{0.025t}\right]$

$\int \cos t\,e^{0.025t} = 40\cos t\,e^{0.025t} + 1600\sin t\,e^{0.025t} - 1600\int \cos t\,e^{0.025t}$

$\int \cos t\,e^{0.025t} + 1600\int \cos t\,e^{0.025t} = 40\cos t\,e^{0.025t} + 1600\sin t\,e^{0.025t}$

$\int \cos t\,e^{0.025t}(1 + 1600) = 40\cos t\,e^{0.025t} + 1600\sin t\,e^{0.025t}$

$\int \cos t\,e^{0.025t} = \dfrac{40\cos t\,e^{0.025t} + 1600\sin t\,e^{0.025t}}{1601}$ —— (*****)

Substitute "$\int \cos t\,e^{0.025t}$" into eq (***)

$= 2000e^{0.025t} + 2000e^{0.025t}\sin t - 2000\left(\dfrac{40\cos t\,e^{0.025t} + 1600\sin t\,e^{0.025t}}{1601}\right)$

$m \cdot e^{0.025t} = 2000e^{0.025t} + 2000e^{0.025t}\sin t - \dfrac{80{,}000\cos t\,e^{0.025t} - 3{,}200{,}000\sin t\,e^{0.025t}}{1601}$

$m = 2000 + 2000\sin t - \dfrac{80{,}000\cos t - 3{,}200{,}000\sin t}{1601} + Ce^{0.025t}$

At time "t" = 0 , m = 150

$$150 = \frac{2000}{1} + \frac{2000\sin(0)}{1} - \frac{80{,}000\cos(0) - 3{,}200{,}000\sin(0)}{1607} + \frac{Ce^{-0.025(0)}}{1}$$

$$150 = \frac{3{,}202{,}000 + 0 - 80{,}000}{1607} + Ce^{-0.025(0)}$$

$$240{,}150 = 3{,}202{,}000 - 80{,}000 + C$$

$$240{,}150 = 3{,}122{,}000 + C$$

$$C =$$

$$150 = 2000 - 49.97 + C$$

$$150 = 1950.03 + C$$

$$C = 150 - 1950.03$$

$$C = -1800.03$$

$$\therefore\ m = 2000 + 2000\sin(t) - \frac{80{,}000\cos(t) - 3{,}200{,}000\sin(t)}{1607} - 1800.03e^{-t/40}$$

(c) MATLAB mfile program solved seperately.

MATLAB R2018a

Editor - C:\Users\Sule Mubarak\Documents\MATLAB\Sule1.m

Sule1.m | Sule2.m

```
% SULE MUBARAK ADEDEJI 18/ENG03/055 CIVIL ENGINEERING %
commandwindow
clear
clc
close all
syms t m(t)
% A = Ordinary differential equation for studying the dynamics of the amount of salt in the tank.
A=diff(m,t)==(50+(50*sin(t))-(0.025*m))
% B = Solved differential equation satisfying the condition of 150lb
% of salt at the initial time.
B = dsolve(A,m(0)==150)
t=0:0.5:450
C=subs(B,t)
plot(t,C)
xlabel('Time(mins)')
ylabel('Amount of salt in the tank(lb)')
title('Dynamic response of the system')
grid on
grid minor
% m = amount of salt in the tank at any time t
```

Command Window

Ln 1 Col 1

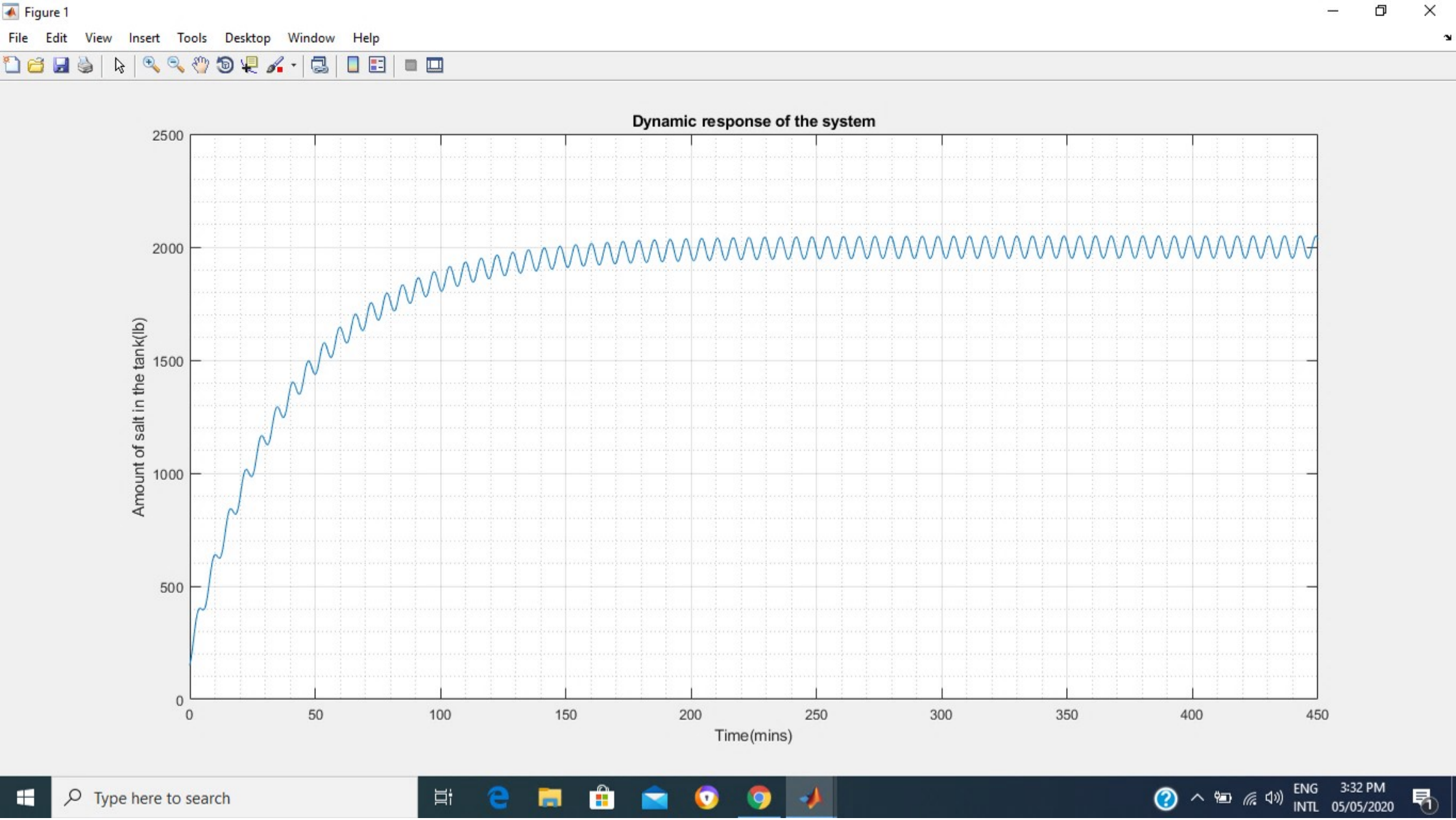
Figure 1
File Edit View Insert Tools Desktop Window Help
Dynamic response of the system
Amount of salt in the tank(lb)
Time(mins)
0
500
1000
1500
2000
2500
0
50
100
150
200
250
300
350
400
450
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MATLAB R2018a

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Sule1.m | Sule2.m

```
% SULE MUBARAK ADEDEJI 18/ENG03/055 CIVIL ENGINEERING %
commandwindow
clear
clc
close all
syms t t1 t2
% Y = Main dynamic model of a system,Ym = Mean dynamic model
t1= 0:2:500 % Even-Numbered time values
t2= 1:2:499 % Odd-Numered time values
Y = (50/0.05)+((50/1.0025)*sin(t2))+((2.5/1.0025)*cos(t2))-((802.49*exp(-0.05*t2)))
Ym = (1000)-(800*exp(-0.05*t1))
t= 0:1:500
y=[Y,Ym]
plot(t,y)
title('Graph of Combined Responses of the system')
xlabel('Time(mins)')
ylabel('Combined responses(Litres)')
grid on
grid minor
Titles={'t(mins)','V(Litres)'}
xlswrite('odevbesdata.xlsx',t(:),'veriler','A2')
xlswrite('odevbesdata.xlsx',y(:),'veriler','B2')
xlswrite('odevbesdata.xlsx',Titles,'veriler','A1')
```

Command Window

script | Ln 7 | Col 1

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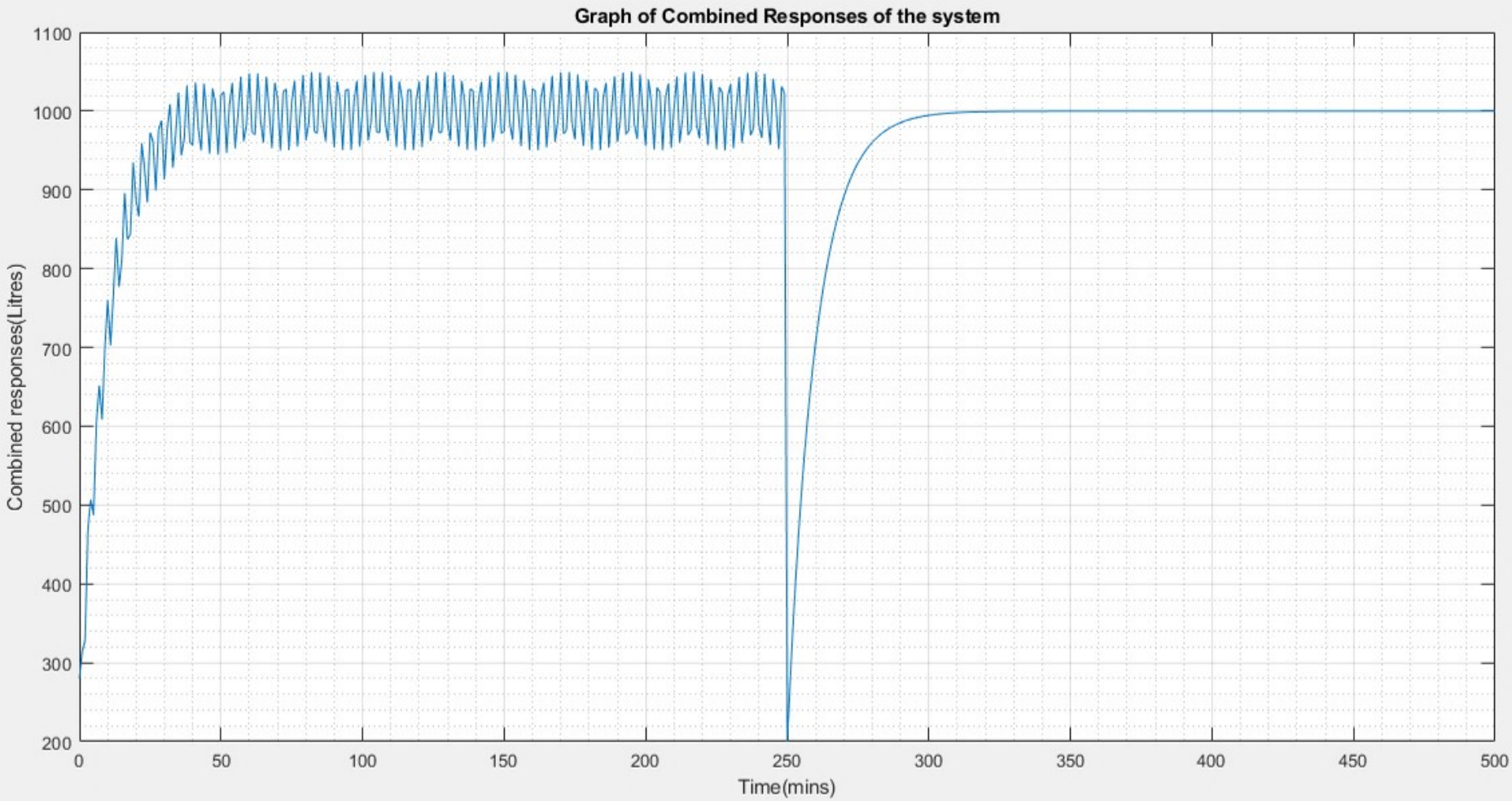
Figure 1

File Edit View Insert Tools Desktop Window Help



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Import - C:\Users\Sule Mubarak\Documents\MATLAB\odevbesdata.xlsx

IMPORT | VIEW

Range: A2:B502
Variable Names Row: 1
SELECTION

Output Type: Table
Text Options
IMPORTED DATA

Replace unimportable cells with NaN
UNIMPORTABLE CELLS

Import Selection
IMPORT

odevbesdata.xlsx

	A	B
	odevbesdata	
	tmins	VLitres
	Number	Number
1	t(mins)	V(Litres)
2	0	279.9639
3	1	313.8601
4	2	327.9009
5	3	469.1423
6	4	506.5922
7	5	487.1398
8	6	604.2824
9	7	651.4694
10	8	608.3676
11	9	699.5850
12	10	759.5410
13	11	702.3679
14	12	765.9535
15	13	838.9333
16	14	776.7953
17	15	811.8028
18	16	895.7197
19	17	836.9388
20	18	843.6308
21	19	934.5610
22	20	886.3180

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