

OKOH ELIJAH EROMOSELE .A.

19/ENG05/048

MECHATRONICS ENGINEERING

MAT102 ASSIGNMENT

~~Algebra~~

Find the point of intersection of the following line on the circle:

1) $x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

let $x - y - 14 = 0 \dots (1)$

$x^2 + y^2 - 6x + 8y = 0 \dots (2)$

from eqn (1) $x - y - 14 = 0$

$\therefore y = x - 14 \dots (3)$

Substitute eqn (3) into eqn (2)

$x^2 + (x - 14)^2 - 6x + 8(x - 14) = 0$

$x^2 + x^2 - 28x + 196 - 6x + 8x - 112 = 0$

$2x^2 - 26x + 84 = 0$

$x^2 - 13x + 42 = 0$

$x^2 - 6x - 7x + 42 = 0$

$x(x - 6) - 7(x - 6) = 0$

$(x - 6)(x - 7) = 0$

$\therefore x = 6 \text{ or } 7$

Recall that $y = x - 14$

\therefore where $x = 6$; $y = 6 - 14$
 $y = -8$

\therefore Co-ordinates of intersection $(6, -8)$

\therefore when $x = 7$; $y = 7 - 14$
 $y = -7$

\therefore Co-ordinates of intersection $(7, -7)$

$$(2) \quad 2x + y - 10 = 0 \quad \dots (1)$$

$$x^2 + y^2 + 4x - 6y = 0 \quad \dots (2)$$

$$\text{from eqn (1) } y = 10 - 2x \quad \dots (3)$$

\therefore substitute eqn (3) into eqn (2)

$$\therefore x^2 + (10 - 2x)^2 + 4x - 6(10 - 2x) = 0$$

$$x^2 + 100 - 40x + 4x^2 + 4x - 60 + 12x = 0$$

$$5x^2 - 24x + 40 = 0$$

$$x_1 = \frac{12}{5} + \frac{2\sqrt{14}}{5}i \quad \text{and} \quad x_2 = \frac{12}{5} - \frac{2\sqrt{14}}{5}i$$

Since the values of x are imaginary, The line and circle do not intersect.

$$(3) \quad x - 5y - 2 = 0 \quad \dots (1)$$

$$x^2 + 25y^2 - 6xy - 16 = 0 \quad \dots (2)$$

$$x = 2 + 5y \quad \dots (3)$$

Substitute eqn (3) into (2)

$$\therefore (2 + 5y)^2 + 25y^2 - 6y(2 + 5y) - 16 = 0$$

$$4 + 20y + 25y^2 + 25y^2 - 12y - 30y^2 - 16 = 0$$

$$50y^2 - 30y^2 + 20y - 12y + 4 - 16 = 0$$

$$20y^2 + 8y - 12 = 0$$

$$5y^2 + 2y - 3 = 0$$

$$5y^2 + 5y - 3y - 3 = 0$$

$$5y(y+1) - 3(y+1) = 0$$

$$(5y - 3)(y + 1) = 0$$

$$\therefore y = -1 \text{ or } 3/5$$

\therefore Recall that $x = 2 + 5y$

$$\text{when } y = -1 \quad \text{when } y = 3/5$$

$$x = 2 + 5(-1) \quad x = 2 + 8(3/5)$$

$$x = 2 - 5 \quad x = 2 + 3$$

$$x = -3 \quad x = 5$$

\therefore The x -ordinates of intersection are $(-3, -1)$ & $(5, 3/5)$,