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MECHANICAL ENGINEERING

19/ENG06/016

SERIAL NO. ; 111

MAT 102 ASSIGNMENT

Find the point of intersection of the following line and the circle:

1. $x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

Solution

$$x - y - 14 = 0 \quad \text{--- (1)}$$

$$x^2 + y^2 - 6x + 8y = 0 \quad \text{--- (2)}$$

From eqn (1);

$$x = y + 14 \quad \text{--- (3)}$$

Substituting eqn (3) into eqn (2)

$$\Rightarrow (y + 14)^2 + y^2 - 6(y + 14) + 8y = 0$$

$$y^2 + 28y + 196 + y^2 - 6y - 84 + 8y = 0$$

$$2y^2 + 30y + 112 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2a

$$y = \frac{-30 \pm \sqrt{(30)^2 - 4 \times 2 \times 112}}{2(2)}$$

2(2)

$$y = \frac{-30 \pm 2}{4}$$

$$\therefore y = -7 \text{ or } -8$$

Substituting $y = -7$ in eqn (3); we have,

$$x = y + 14$$

$$x = -7 + 14$$

$$x = 7$$

\therefore One of the points of intersection is $(7, -7)$

Substituting $y = -8$ in eqn (3); we have,

$$x = y + 14$$

$$x = -8 + 14$$

$$x = 6$$

\therefore The other point of intersection is $(6, -8)$

$$2. \quad 2x + y - 10 = 0 \text{ and } x^2 + y^2 + 4x - 6y = 0$$

Solution

$$2x + y - 10 = 0 \quad \text{--- (1)}$$

$$x^2 + y^2 + 4x - 6y = 0 \quad \text{--- (2)}$$

From eqn (1);

$$y = 10 - 2x \quad \text{--- (3)}$$

Substitute eqn (3) in eqn (2)

$$\Rightarrow x^2 + (10 - 2x)^2 + 4x - 6(10 - 2x) = 0$$

$$x^2 + 4x^2 - 40x + 100 + 4x - 60 + 12x = 0$$

$$5x^2 - 24x + 40 = 0$$

x is an imaginary number so the line doesn't intersect.

3. $x - 5y - 2 = 0$ and $x^2 + 25y^2 - 6xy - 16 = 0$

Solution

$$x - 5y - 2 = 0 \quad \text{--- (1)}$$

$$x^2 + 25y^2 - 6xy - 16 = 0 \quad \text{--- (2)}$$

From eqn (1);

$$x = 5y + 2 \quad \text{--- (3)}$$

Substituting eqn (3) into eqn (2)

$$(5y + 2)^2 + 25y^2 - 6y(5y + 2) - 16 = 0$$

$$25y^2 + 20y + 4 + 25y^2 - 30y^2 - 12y - 16 = 0$$

$$20y^2 + 8y - 12 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 20 \times (-12)}}{2(20)}$$

$$y = \frac{-8 \pm 32}{40}$$

$$\therefore y = -1 \text{ or } \frac{3}{5}$$

Substituting $y = -1$ in eqn (3); we have,

$$x = 5y + 2$$

$$x = 5(-1) + 2$$

$$x = -3$$

\therefore One of the points of intersection is $(-3, -1)$

Substituting $y = \frac{3}{5}$ in eqn (3); we have,

$$x = 5y + 2$$

$$x = 5\left(\frac{3}{5}\right) + 2$$

$$x = 5$$

\therefore The other point of intersection is $(5, \frac{3}{5})$