NAME: OJO-ONI DANIEL OLUWASEGUN, MATRIC NO.; 19/ENG09/016, DEPARTMMENT; AERONAUTICAL ENGINEERING, COURSE, MAT 102 GENERAL MATHEMATICS II, LECTURER; MR. OKUNLOLA

NAME: OJO-ONI DANIEL OLUWASEGUN, MAT 102, GENERAL MATH II
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ASSIGNMENT FOR MR. OKUNLOLA. MATRIC NO: 19/ENG09/016.

QUESTIONS AND ANSWERS:

1) If $\bar{M} = Pi - 6j - 3k$, $\bar{N} = 4i + 3j - k$, $\bar{O} = i - 3j + 2k$, find the value of P for which:

a) $\bar{M}$ and $\bar{N}$ are perpendicular to each other

Solution

Given $\bar{M} = Pi - 6j - 3k$ and $\bar{N} = 4i + 3j - k$

For perpendicular vectors, $\bar{M} \cdot \bar{N} = 0$.

$\therefore (Pi - 6j - 3k) \cdot (4i + 3j - k) = 0$.

$4P - 18 + 3 = 0$

$\therefore 4P - 15 = 0$, $4P = 15$, $\therefore P = 15/4$ for perpendicular vectors.

b) $\bar{M}$, $\bar{N}$ and $\bar{O}$ are co-planar:

Solution.

Co-planar vectors or parallel vectors occurs when $\bar{M} \cdot (\bar{N} \times \bar{O}) = 0$.

Since $\bar{M} = Pi - 6j - 3k$, $\bar{N} = 4i + 3j - k$, $\bar{O} = i - 3j + 2k$.

$$\therefore \bar{M} \cdot (\bar{N} \times \bar{O}) = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} \end{array} = 0$$

$$= P \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$= P(-6 - 3) + 6(8 + 1) - 3(-12 - 3) = 0$

$= P(-9) + 6(9) - 3(-15) = 0$

$= -9P + 54 + 45 = 0$

$= -9P + 99 = 0$, $\therefore -9P = -99$

$\therefore P = 11$ when $\bar{M}$, $\bar{N}$ and $\bar{O}$ are parallel.

2) Find the direction cosines and the unit vector along the sum

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of $3i+2j+5k$, $2i-j+6k$ and $5i+2j-3k$.

Sol.

let $\bar{r}$ be the sum of $(3i+2j+5k)$, $(2i-j+6k)$ and $(5i+2j-3k)$

$\therefore \bar{r} = (3+2+5)i + (2-1+2)j + (5+6-3)k$.

$\therefore \bar{r} = 10i + 3j + 8k$.

a). The direction Cosine is given as $\cos\alpha$, $\cos\beta$, $\cos\gamma$

$|r| = \sqrt{10^2+3^2+8^2} = \sqrt{173} = 13.15$.

$a_x = 10$, $a_y = 3$, $a_z = 8$.

$\therefore \cos\alpha = \frac{a_x}{|r|} = \frac{10}{13.15} = 0.7605$.

$\cos\beta = \frac{a_y}{|r|} = \frac{3}{13.15} = 0.2281$.

$\cos\gamma = \frac{a_z}{|r|} = \frac{8}{13.15} = 0.6084$.

b) The Unit Vector $= \frac{\bar{r}}{|r|} = \frac{10i+3j+8k}{\sqrt{173}}$.

$\therefore$ The Unit Vector is $= \frac{10}{\sqrt{173}}i + \frac{3}{\sqrt{173}}j + \frac{8}{\sqrt{173}}k$

3). If $F = 3ui + u^2j + (u+2)k$ and $V = 2ui - 3uj + (u-2)k$ evaluate the Integral of $(F\times V)du$ from 0 to 1.

Sol.

Given $F = 3ui + u^2j + (u+2)k$ and

$V = 2ui - 3uj + (u-2)k$.

$$(F\times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & u+2 \\ 2u & u-2 \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i\left(u^3-2u^2-(-3u^2-6u)\right) - j\left(3u^2-6u-(2u^2+4u)\right) + k\left(-9u^2-2u^3\right)$$

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$$= i(u^3-2u^2+3u^2+6u) - j(3u^2-6u-2u^2-4u) + (-9u^2-2u^3)k$$

$$= i(u^3+u^2+6u) - j(u^2-10u) + (-9u^2-2u^3)k$$

$$=(u^3+u^2+6u)i - (u^2-10u)j + (-9u^2-2u^3)k = F\times V$$

$\therefore$

Given $(F\times v) = (u^3+u^2+6u)i - (u^2-10u)j + (-9u^2-2u^3)k$.

$$\int_0^1 (F\times v)\,du = \int_0^1 (u^3+u^2+6u)i - (u^2-10u)j + (-9u^2-2u^3)k$$

Therefore, integrating

$$= i\left[\frac{u^{3+1}}{4}+\frac{u^{2+1}}{3}+\frac{6u^2}{2}\right]_0^1 - j\left[\frac{u^3}{3}-\frac{10u^2}{2}\right]_0^1 + k\left[-\frac{9u^3}{3}-\frac{2u^4}{4}\right]_0^1$$

Simplifying

$$= i\left[\frac{u^4}{4}+\frac{u^3}{3}+3u^2\right]_0^1 - j\left[\frac{u^3}{3}-5u^2\right]_0^1 + k\left[-3u^3-\frac{u^4}{2}\right]_0^1$$

$$= i\left[\left(\frac{1^4}{4}+\frac{1^3}{3}+3\right)-\left(\frac{0^4}{4}+\frac{0^3}{3}+0\right)\right] = i\left[\frac{1}{4}+\frac{1}{3}+3\right] = \frac{43}{12}i$$

$$= j\left[\left(\frac{1^3}{3}-5\right)-\left(\frac{0^3}{3}-0\right)\right] = j\left[\frac{1}{3}-5\right] = -\frac{14}{3}j$$

$$= k\left[\left(-3(1)^3-\frac{1^4}{2}\right)-(0-0)\right] = k\left(-3-\frac{1}{2}\right) = -\frac{7}{2}k$$

$\therefore$ Combining it,

$$\int_0^1 (F\times v)\,du = \frac{43}{12}i - \frac{14}{3}j - \frac{7}{2}k$$