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1. Find the point of intersection of the following line on the circle.

i. $x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

$y = x - 14$

$(x^2) + (x - 14)^2 - 6x + 8(x - 14) = 0$

$x^2 + x^2 - 28x + 196 - 6x + 8x - 112 = 0$

$2x^2 - 26x + 84 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-26 \pm \sqrt{(-26^2) - 4 \times 2 \times 84}}{2 \times 2}$

$= \frac{-26 \pm 2}{4}$

$x = \frac{-26 + 2}{4}$ or $x = \frac{-26 - 2}{4}$

$x = -6$ or $x = -7$

$\therefore y = -6 - 14$ or $y = -7 - 4$
$= -20$ $= -11$

Points of intersection
$(-6, -20)$ and $(-7, -11)$.

ii. $2x + y - 10 = 0$ and $x^2 + y^2 + 4x - 6y = 0$

$y = -2x + 10$

$x^2 + (-2x + 10)^2 + 4x - 6(-2x + 10) = 0$

$x^2 + 4x^2 - 40x + 100 + 4x + 12x - 60 = 0$

$5x^2 - 24x + 40 = 0$

After using the quadratic equation to find

$x = \frac{12}{5} + \frac{2\sqrt{14}}{5}$ and $x = \frac{12}{5} - \frac{2\sqrt{14}}{5}$

$y = -2\left(\frac{12}{5} + \frac{2\sqrt{14}}{5}\right) + 10$ and $y = -2\left(\frac{12}{5} - \frac{2\sqrt{14}}{5}\right) + 10$

$y = \frac{26 - 4\sqrt{14}}{5}$ and $y = \frac{26 + 4\sqrt{14}}{5}$

Points of intersection

$\left(\frac{12}{5} + \frac{2\sqrt{14}}{5}, \frac{26 - 4\sqrt{14}}{5}\right)$ and $\left(\frac{12}{5} - \frac{2\sqrt{14}}{5}, \frac{26 + 4\sqrt{14}}{5}\right)$

iii) $x - 5y - 2 = 0$ and $x^2 + 25y^2 - 6xy - 16 = 0$

$y = \frac{x - 2}{5}$

$x^2 + 25\left(\frac{x-2}{5}\right)^2 - 6x\left(\frac{x-2}{5}\right) - 16 = 0$

$x^2 + 25\left(\frac{x^2 - 4x + 4}{25}\right) - \frac{6x^2 + 12x}{5} - 16 = 0$

$\cancel{5}x^2 + x^2 - 4x + 4 - \frac{6x + 12x}{\cancel{5}} - 16 = 0 \times 5$

$2x^2 + 2x - 12 = 0$

After factorization

$x = 2$ and $x = -3$

$y = \frac{2-2}{5} = 0$ and $x = \frac{-3-2}{5} = \frac{-5}{5} = -1$

Points of intersection

$(2, 0)$ and $(-3, -1)$