

Name: Mathios Shadrach Ojochogbe  
DEPT: Mechanical Engineering  
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MAT 102

### Assignment

Find the Point of Intersection of the following line on the Circle

1.  $x - y - 14 = 0$  and  $x^2 + y^2 - 6x + 8y = 0$

Solution

$$x - y - 14 = 0$$

$$y = x - 14$$

Substituting for  $y$  in the circle

$$x^2 + (x - 14)^2 - 6x + 8(x - 14) = 0$$

$$x^2 + x^2 - 14x - 14x + 196 - 6x + 8x - 112 = 0$$

$$2x^2 - 26x + 84 = 0$$

$$x^2 - 13x + 42 = 0$$

Using:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4(1)(42)}}{2}$$

$$x = \frac{13 \pm 1}{2}$$

$$x = \frac{14}{2} = 7 \quad \text{and} \quad x = \frac{12}{2} = 6$$

Substituting for  $x$  in the line

$$\text{When } x = 7; \quad y = 7 - 14 = -7$$

$$\text{When } x = 6; \quad y = 6 - 14 = -8$$

$\therefore$  The Points of Intersection of the line on the circle are:  $(7, -7)$  and  $(6, -8)$

2.  $2x + y - 10 = 0$  and  $x^2 + y^2 + 4x - 6y = 0$

Solution

$$2x + y - 10 = 0$$

$$y = -2x + 10$$

Substituting for  $y$  in the circle

$$x^2 + (-2x + 10)^2 + 4x - 6(-2x + 10) = 0$$

$$x^2 + 4x^2 - 20x - 20x + 100 + 4x + 12x - 60 = 0$$

$$5x^2 - 24x + 40 = 0$$

Using:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{24 \pm \sqrt{(-24)^2 - 4(5)(40)}}{2(5)}$$

$$x = \frac{12}{5} + \frac{2\sqrt{14}}{5}; \quad \text{and} \quad x = \frac{12}{5} - \frac{2\sqrt{14}}{5}$$

$$x = 3.4$$

$$x = 1.4$$

When  $x = 3.4$ ,  $y = -2x + 10 = 3.2$

When  $x = 1.4$ ,  $y = -2(1.4) + 10 = 7.2$

$\therefore$  The points of intersection of the line on the circle are  $(3.4, 3.2)$  and  $(1.4, 7.2)$

3.  $x - 5y - 2 = 0$  and  $x^2 + 25y^2 - 6xy - 16 = 0$

Solution

$$y = \frac{x-2}{5}$$

Substitute for  $y$  in the circle  $x^2 + 25y^2 - 6xy - 16 = 0$

$$x^2 + 25\left(\frac{x-2}{5}\right)^2 - 6x\left(\frac{x-2}{5}\right) - 16 = 0$$

$$x^2 + \left(\frac{25x^2}{5} - \frac{25 \times 2}{5}\right) - \left(\frac{6x \times x}{5} - \frac{6x \times 2}{5}\right) - 16 = 0$$

$$x^2 + (5x^2 - 10) - \left(\frac{6x^2}{5} - \frac{12x}{5}\right) - 16 = 0$$

$$x^2 + 25x^2 - 10x + 10 - \frac{6x^2}{5} + \frac{12x}{5} - 16 = 0$$

$$5x^2 + 125x^2 - 500x + 500 - 6x^2 + 12x - 16 = 0$$

$$130x^2 - 500x + 500 - 6x^2 + 12x - 16 = 0$$

$$124x^2 - 488x + 484 = 0$$

Using:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{488 \pm \sqrt{(-488)^2 - 4(124)(484)}}{2(124)}$$

$$x = \frac{61 + \sqrt{30}}{31} ; \quad \text{and} \quad x = \frac{61 - \sqrt{30}}{31} ;$$

$$x = 2.15$$

$$x = 1.79$$

$$\text{When } x = 2.15 ; \quad y = \frac{2.15 - 2}{5} = 0.03$$

$$\text{When } x = 1.79 ; \quad y = \frac{1.79 - 2}{5} = -0.042$$

$\therefore$  The Points of intersection of the line on the circle are  $(2.15, 0.03)$  and  $(1.79, -0.042)$