MAT 102
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MECHATRONICS ENGINEERING
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1 If $M = pi - 6j - 3k$ $N = 4i + 3j - k$, $O = i - 3j + 2k$
find the value of p for which

a) M and N are perpendicular

$A \cdot B = 0$

$M \cdot N = 0$

$(pi - 6j - 3k) \cdot (4i + 3j - k) = 0$

$4p - 18 + 3 = 0$

$4p - 15 = 0$

$4p = 15$

$p = 15/4 = \underline{\underline{3.75}}$

b M, N and O are coplanar

$A \cdot (B \times C) = 0$

$M \cdot (N \times O) = 0$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$$p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} -3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$p(6-3) + 6(8+1) - 3(-12-3) = 0$

$3p + 54 + 45 = 0$

$3p + 99 = 0$

$3p = -99$

$p = \underline{\underline{-33}}$

2 find the direction cosines and the unit vector along the sum of $3i + 2j + 5k$, $2i - j + 6k$ and $5i + 2j - 3k$

$r = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$
$r = 10i + 3j + 8k$

Direction cosines

$\cos\alpha = \frac{x}{|r|}$ $\cos\beta = \frac{y}{|r|}$ $\cos r = \frac{z}{|r|}$

$|r| = \sqrt{10^2 + 3^2 + 8^2}$

$|r| = \sqrt{173}$

$\cos\alpha = \frac{10}{\sqrt{173}} = 0.76286$

$\cos\beta = \frac{3}{\sqrt{173}} = 0.2281$

$\cos r = \frac{8}{\sqrt{173}} = 0.60823$

The unit vector

$e_r = \frac{r}{|r|}$

$e_r = \frac{10i + 3j + 8k}{\sqrt{173}}$

3 If $f = 3ui + u^2 j + (u+2)k$ and
$v = 2ui - 3uj + (u-2)k$ evaluate the integral of $(f \times v)\,du$ from 0 to 1

$$(f \times v) = \begin{vmatrix} i & j & k \\ 3u & u^2 & u+2 \\ 2u & -3u & u-2 \end{vmatrix}$$

$$i\begin{vmatrix} u^2 & u+2 \\ -3u & u-2 \end{vmatrix} - j\begin{vmatrix} 3u & u+2 \\ 2u & u-2 \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$i\left(u^3 - 2u^2 - (-3u^2 - 6u)\right) - j\left(3u^2 - 6u - (2u^2 + 4u)\right)$$
$$+ k\left(-9u^2 - (2u^3)\right)$$

$i(u^3 - 2u^2 + 3u^2 + 6u) - j(3u^2 - 6u - 2u^2 - 4u) + k(-9u^2 - 2u^3)$

$i(u^3 + u^2 + 6u) - j(u^2 - 10u) + k(-9u^2 - 2u^3)$

$i(u^3 + u^2 + 6u) + j(-u^2 + 10u) + k(-9u^2 - 2u^3)$

$$\int_0^1 (u^3 + u^2 + 6u)i + (-u^2 + 10u)j + (-9u^2 - 2u^3)k \, du$$

$$\left(\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right)i + \left(\frac{-u^3}{3} + \frac{10u^3}{2}\right)j + \left(\frac{-9u^3}{3} - \frac{2u^4}{4}\right)k \Bigg|_0^1$$

$$\left(\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right)i + \left(\frac{-u^3}{3} + 5u^2\right)j + \left(-3u^3 - \frac{u^4}{2}\right)k \Bigg|_0^1$$

$$\left(\frac{1}{4} + \frac{1}{3} + 3(1)\right)i + \left(\frac{-(1)^3}{3} + 5(1)^2\right)j + \left(-3(1)^3 - \frac{(1)^4}{2}\right)k$$

$$\left(\frac{1}{4} + \frac{1}{3} + 3\right)i + \left(\frac{-1}{3} + 5\right)j + \left(-3 - \frac{1}{2}\right)k$$

$$\frac{43}{12}i + \frac{14}{3}j - \frac{7}{2}k$$

$$\underline{\underline{43i + 56j - 42k}}$$