ISAAC ENE GRACE 19/ENG08/007

BIOMEDICAL ENG.

(1) $x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

The equation of the center of the circle

given by $x^2 + y^2 - 6x + 8y = 0$

Rewrites $(x-3)^2 - 9 + (y+4)^2 - 16 = 0$

$(x-3)^2 + (y+4)^2 - 16 - 9 = 0$

$(x-3)^2 + (y+4)^2 - 25 = 0$

$(x-3)^2 + (y+4)^2 = 25$

Centre = $(3, -4)$, radius = $\sqrt{25} = 5$.

∴ linear equation, $x - y - 14 = 0$, make y sub of the formula

$y = x - 14$ — (ii) subt (ii) into (i)

$(x-3)^2 + (y+4)^2 = 25$ — (i)

$(x-3)(x-3) + (x-14+4)^2 = 25$

$x(x-3) - 3(x-3) + (x-10)^2 = 25$

$x^2 - 3x - 3x + 9 + (x-10)^2 = 25$

$x^2 - 6x + 9 + x^2 - 20x + 10 = 25$

$2x^2 - 26x + 84 = 0$ factorizing

$(2x^2 - 14x)(-12x + 84) = 0$

$2x(x-7) - 12(x-7) = 0$

$(2x - 12)(x - 7) = 0$

$x = 6$, or $x = 7$

subst the values of x into the linear

for $x = 6$ ∴ $y = x - 14 = 6 - 14 = -8$

for $x = 7$ ∴ $y = 7 - 14 = -7$

$A(6, -8)$

$B(7, -7)$

$\therefore A(6,-8), B(7,-7)$ are the points of Intersection.

b) $2x + y - 10 = 0$ and $x^2 + y^2 + 4x - 6y = 0$.

Soln.

Given $x^2 + y^2 + 4x - 6y = 0$, re-arranging

$x^2 + 4x + y^2 - 6y = 0$.

$(x+2)^2 - 4 + (y-3)^2 - 9 = 0$. Collect like terms.

$(x+2)^2 + (y-3)^2 - 13 = 0$. $\therefore (x+2)^2 + (y-3)^2 = 13 \cdots$ (i).

Centre $= (-2, 3)$ and radius $= \sqrt{13}$.

linear equation $= 2x + y - 10 = 0 \cdots$ (ii).

$\therefore y = 10 - 2x$. (iii) substitute (iii) into (i)

From (i), $(x+2)^2 + (y-3)^2 = 13$

$(x+2)^2 + (10-2x-3)^2 = 13$.

$\therefore x^2 + 4x + 4 + (7-2x)^2 = 13$

$x^2 + 4x + 4 + (7(7-2x) - 2x(7-2x)) = 13$.

Simplifying, $x^2 + 4x + 4 + (49 - 14x - 14x + 4x^2) = 13$

$x^2 + 4x + 4 + (49 - 28x + 4x^2) = 13$.

$5x^2 - 24x + 40 = 0$.

when we solve this, the value of x gives an imaginative number i.e $x = \frac{12}{5} + \frac{2\sqrt{14}}{5}i$.

This makes it quite Impossible to find the points of the circle and the line.

c) $x - 5y - 2 = 0$ and $x^2 + 25y^2 - 6xy - 16 = 0$

Soln.

Given $x^2 + 25y^2 - 6xy - 16 = 0$. to be the equation,

$x^2 + y(25y - 6x) - 16 = 0 \cdots$ (i).

Given $x - 5y - 2 = 0$

$x = 5y - 2$ (ii) substituting.

$(5y-2)^2 + y(25y - 6(5y-2)) - 16 = 0$. expanding.

$25y^2 - 20y + 4 + y(25y - 30y + 12) - 16 = 0$.

$25y^2 - 20y + 25y^2 - 30y^2 + 12y - 12 = 0$

$20y^2 - 8y - 12 = 0$. solving quadratically.

$(20y^2 - 20y) + (12y - 12) = 0.$

$20y(y-1) + 12(y-1) = 0.$

$(20y + 12)(y-1) = 0.$

$\therefore 20y + 12 = 0 \quad \text{or} \quad y = 1.$

$\frac{20y}{20} = \frac{-12}{20} \quad \text{or} \quad y = 1.$

$y = -3/5 \quad \text{or} \quad y = 1.$

$\therefore$ using the Values above to find the points of intersection,

when $y = -3/5$, from (ii), $x = 5y - 2 = 5\left(\frac{-3}{5}\right) - 2 = -5.$

$\therefore$ Point $A = (-5, -3/5)$

when $y = 1$, from (ii), $x = 5y - 2 = 5(1) - 2 = 3$

Point $B = (3, 1).$

$\therefore$ The points will be $A(-5, -3/5)$ and $B(3, 1).$