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CHEMICAL ENGINEERING.
18/ENG 01/016.
ENGINEERING MATHEMATICS.

1. Mixing,
1200 gal of water } initially.
150 lb of salt }

50 gal of brine (salt & water), each contain $(1 + 5 \sin t)$ lb of salt,

30 gal/min removes from the tank.

Accumulation rate within a system;

INPUT rate into the system - OUTPUT rate from the system.

$$\frac{dy}{dt} = y_{in} - y_{out} \quad ; \quad \frac{dm}{dt} = m_{in} - m_{out}$$

Since 30 gal removes per minute, and one gallon contains $(1 + 5 \sin t)$ lb of salt, it means that the amount of salt leaving the tank is

$$m_{out} = 30 \frac{\text{gal}}{\text{min}} \times (1 + 5 \sin t) \frac{\text{lb}}{\text{gal}}$$

$$m_{out} = 30 (1 + 5 \sin t) \text{ lb/min.}$$

The tank contains 1200 gal of water with dissolved salt, and 30 gal of the solution leaves the tank per minute,

$\frac{30 \text{ gal}}{1200 \text{ gal}} = 0.025 = 2.5\%$ of the content of the tank. If that is the case,

2.5% of salt present in the tank will also leave the tank per minute, therefore,

$$m_{out} = 2.5\% \text{ of } m.$$

$$\text{from } \frac{dm}{dt} = M_{in} - M_{out}$$

$$\frac{dm}{dt} = 50(1 + \sin t) - 2.5\% \text{ of } m.$$

$$\frac{dm}{dt} = 50(1 + \sin t) - 0.025m$$

$$b.) \frac{dm}{dt} = 50(1 + \sin t) - 0.025m.$$

$$\frac{dm}{dt} + 0.025m = 50(1 + \sin t)$$

using the linear equation method,

$$\frac{dy}{dx} + py = Q,$$

$$\frac{dm}{dt} + pm = Q, \text{ where } p = 0.025, Q = 50(1 + \sin t)$$

$$\therefore \int p dt = 0.025t.$$

$$I.F = e^{\int p dt}$$

$$I.F = e^{0.025t}$$

$$\therefore m \cdot I.F = \int Q \cdot I.F \cdot dt$$

$$m e^{0.025t} = \int 50(1 + \sin t) e^{0.025t} \cdot dt.$$

$$m e^{0.025t} = 50 \int e^{0.025t} \cdot dt + 50 \int e^{0.025t} \sin t \cdot dt.$$

$$m e^{0.025t} = 50 \cdot \frac{e^{0.025t}}{0.025} + 50 \int e^{0.025t} \sin t \cdot dt.$$

$$\therefore \text{To integrate } \int e^{0.025t} \sin t \cdot dt,$$

we'll use integration part,
 $\int u dv = uv - \int v du$

$$\therefore u = e^{0.025t} \quad dv = \sin t$$

$$du = 0.025 e^{0.025t} \quad v = -\cos t$$

$$\therefore \int e^{0.025t} \sin t = e^{0.025t} \cdot -\cos t - \int -\cos t \cdot 0.025 e^{0.025t}$$

$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t - \int -\cos t \cdot 0.025 e^{0.025t}$$

$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t + 0.025 \int e^{0.025t} \cos t$$

using integration by part for $\int e^{0.025t} \cos t$,

$$u = e^{0.025t} \quad dv = \cos t$$

$$du = 0.025 e^{0.025t} \quad v = \sin t$$

$$\therefore \int e^{0.025t} \cos t = e^{0.025t} \sin t - \int \sin t \cdot 0.025 e^{0.025t}$$

$$= e^{0.025t} \sin t - 0.025 \int \sin t e^{0.025t}$$

Therefore,

$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t + 0.025 \left[e^{0.025t} \sin t - 0.025 \int \sin t e^{0.025t} \right]$$

$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t + 0.025 \left[e^{0.025t} \sin t - 0.025 \int e^{0.025t} \sin t \right]$$

$$\text{Let } W = \int e^{0.025t} \sin t$$

$$\therefore W = -e^{0.025t} \cos t + 0.025 \left[e^{0.025t} \sin t - 0.025 W \right]$$

$$W = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t - 6.25 \times 10^{-4} W$$

$$W + 6.25 \times 10^{-4} W = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t$$

$$1.000625 W = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t$$

$$1.000625 W = -e^{0.025t} (\cos t - 0.025 \sin t)$$

$$W = \frac{-e^{0.025t} (\cos t - 0.025 \sin t)}{1.000625}$$

$$\text{Recall } W = \int e^{0.025t} \sin t$$

$$\therefore \int e^{0.025t} \sin t = \frac{-e^{0.025t} (\cos t - 0.025 \sin t)}{1.000625} + C$$

from equation * * *

$$me^{0.025t} = \frac{50 \cdot e^{0.025t}}{0.025} + 50 \int e^{0.025t} \sin t \, dt$$

Therefore,

$$me^{0.025t} = 50 \cdot \frac{e^{0.025t}}{1.000625} + 50 \left[\frac{-e^{0.025t}}{1.000625} (\cos t - 0.025 \sin t) + C \right]$$

$$me^{0.025t} = 2000 \frac{e^{0.025t}}{1.000625} - 50 \cdot \frac{e^{0.025t}}{1.000625} (\cos t - 0.025 \sin t) + 50C$$

divide through by $e^{0.025t}$.

$$\therefore m = 2000 - \frac{50}{1.000625} (\cos t - 0.025 \sin t) + \frac{50}{e^{0.025t}} C.$$

\therefore when $m = 150$;
 $t = 0$.

$$150 = 2000 - \frac{50}{1.000625} (\cos(0) - 0.025 \sin(0)) + \frac{50}{e^{0.025(0)}} C$$

$$150 = 2000 - 49.9687(1 - 0) + \frac{50}{e^0} C.$$

$$150 = 2000 - 49.9687 + 50C$$

$$150 = 1950.0313 + 50C.$$

$$150 - 1950.0313 = 50C.$$

$$-1800.0313 = 50C.$$

$$C = \frac{-1800.0313}{50}$$

$$C = -36.000626.$$

$$C \approx -36.$$

\therefore

$$m = 2000 - \frac{50}{1.000625} (\cos t - 0.025 \sin t) + \frac{50}{e^{0.025t}} (-36).$$

$$m = 2000 - 49.9687 (\cos t - 0.025 \sin t) + \frac{(-1800)}{e^{0.025t}}$$

$$m = 2000 - 49.9687 (\cos t - 0.025 \sin t) - \frac{1800}{e^{0.025t}}.$$

c) MATLAB mfile program;

commandwindow

clear

clc

close all

syms m(t)

mout = 0.025 * m.

min = 50 * (1 + sin(t)).

m = dsolve(diff(m,t) == min - mout, m(0) == (50))

step_hour = 0.5/60

t = 0:step_hour:7.5

m_values = subs(m, t * 60).

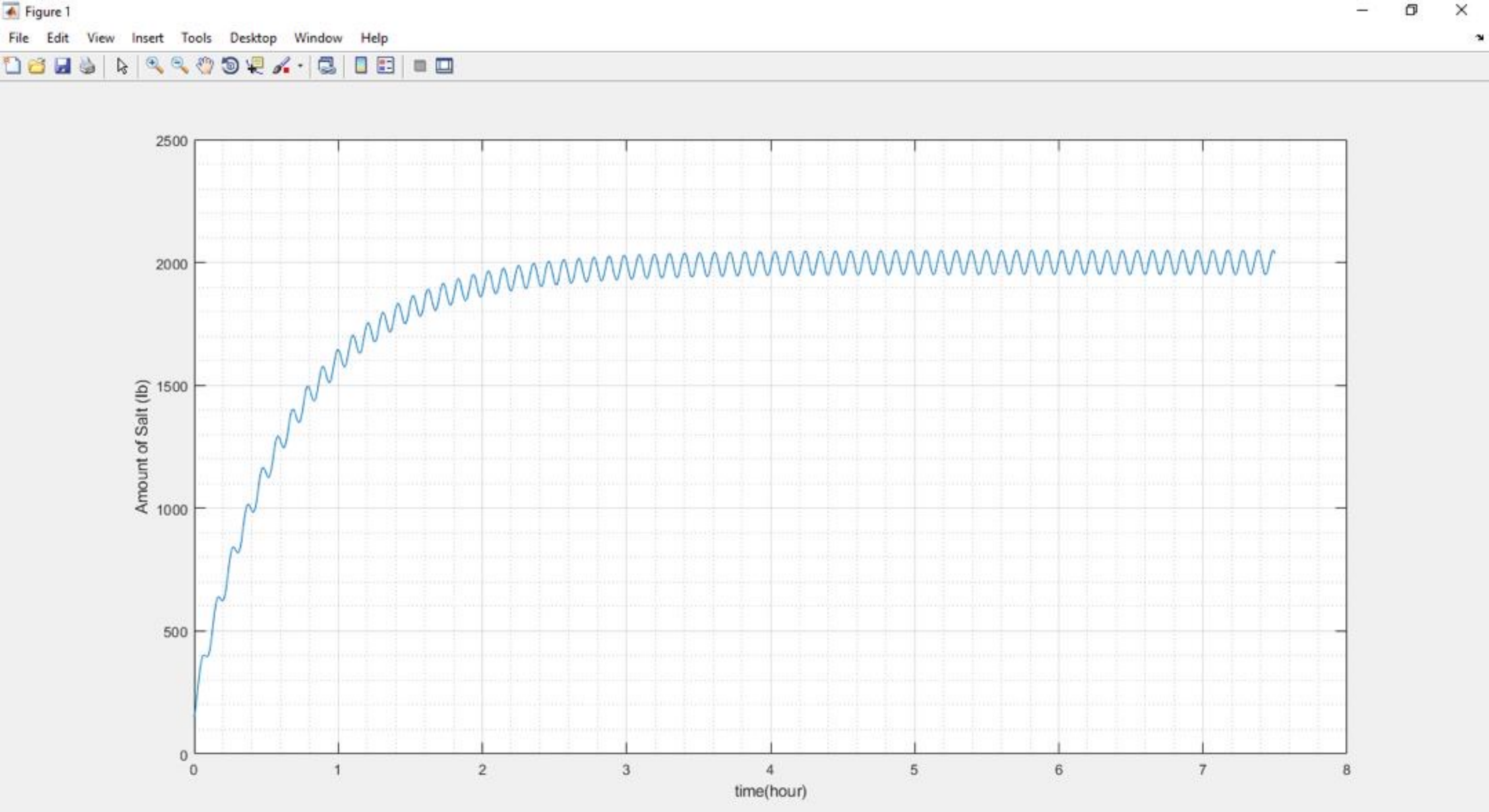
plot(t, m_values)

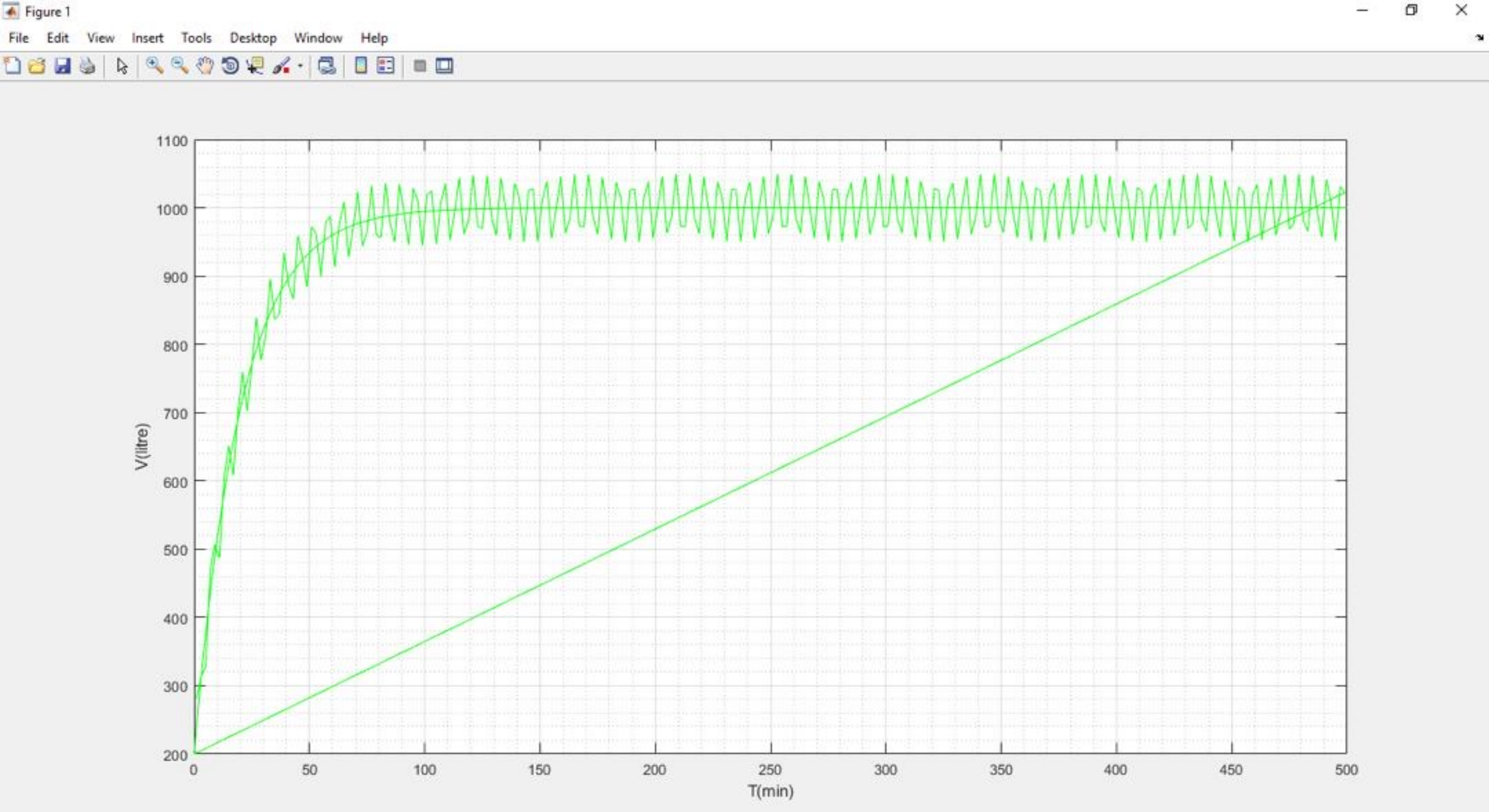
grid on

grid minor

xlabel('time (hour)')

ylabel('Amount of salt (lb)').





Clipboard: Paste, Cut, Copy, Format Painter

Font: Calibri, 11, Bold, Italic, Underline, Text Color, Background Color

Alignment: Wrap Text, Merge & Center, Left, Center, Right, Indent

Number: General, Currency, Percentage, Decimals, Thousands Separator

Styles: Conditional Formatting, Format as Table, Cell Styles

Cells: Insert, Delete, Format

Editing: AutoSum, Fill, Clear, Sort & Filter, Find & Select

D1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	t(min)	V(Litre)																			
2	1	279.9639																			
3	3	313.8601																			
4	5	327.9009																			
5	7	469.1423																			
6	9	506.5922																			
7	11	487.1398																			
8	13	604.2824																			
9	15	651.4694																			
10	17	608.3676																			
11	19	699.585																			
12	21	759.541																			
13	23	702.3679																			
14	25	765.9535																			
15	27	838.9333																			
16	29	776.7953																			
17	31	811.8028																			
18	33	895.7197																			
19	35	836.9388																			
20	37	843.6308																			
21	39	934.561																			
22	41	886.318																			
23	43	866.4219																			
24	45	959.1673																			
25	47	927.1558																			