① Applying the balance law

Accumulation rate of salt within a system = Input rate of salt into the system − Output rate of salt from the system (1)

Denoting the amount of salt present in the tank at any time t as y, its time rate of change is given as

$$\frac{dy}{dt} = y_{in} - y_{out}$$

Since 50 gal of brine enter the tank per minute and one gallon contains $(1+\sin t)$ lb of salt;

(i) at t = 1; $(1+\sin t) = (1+\sin(1)) = 1.02$ lb of salt;

∴ it means that the amount of salt entering the tank is

$$y_{in} = 50\,\frac{gal}{min} \times 1.02\,\frac{lb}{gal} = 51\,\frac{lb}{min}$$

The tank contains 1200 gal of water with the dissolved salt, & ~~50~~ 30 gallons of the solution leaves the tank per minute. That is $\frac{30\,gal}{1200\,gal} = 0.025 = 2.5\%$ of the content of the tank. If that is the case, 2.5% of the salt present in the tank will also leave the tank per minute. In other words,

$y_{out} = 2.5\%$ of y.

ⓐ

Therefore; $\frac{dy}{dt}\frac{lb}{min} = 51\frac{lb}{min} - 2.5\%\text{ of } y\,\frac{lb}{min}$

ⓑ $\frac{dy}{dt} = 51 - 0.025y$; $\frac{dy}{dt} = -0.025y + 51$;

$$\frac{dy}{dt} = -0.025\left[\frac{-0.025y}{-0.025y} + \frac{51}{-0.025}\right];\ \frac{dy}{dt} = -0.025(y - 2040);$$

$$\frac{dy}{(y-2040)} = -0.025dt; \int \frac{dy}{(y-2040)} = \int -0.025dt;$$

$$\int \frac{dy}{(y-2040)} = -0.025\int dt; \ln(y-2040) = -0.025t + C;$$

$$y-2040 = e^{-0.025t+C}; \quad y-2040 = e^{-0.025t}e^{C};$$

$$y-2040 = e^{-0.025t}y_0; \quad y-2040 = y_0e^{-0.025t};$$

$y = y_0e^{-0.025t} + 2040$; Given that when $t=0$ min (initially), $y = 150$ lb;

$$150 = y_0e^{-0.025t(0)} + 2040; \quad 150 - 2040 = y_0 \times 1;$$

$y_0 = -1890$

So;

$$y = -1890e^{-0.025t} + 2040$$

$$y = 2040 - 1890e^{-0.025t}$$ //