MATHS 104

OSOBA VICTORIA ETHIGOCHO
MBBS
19/MHS01/280

1) $\int \frac{11-3x}{x^2+2x-3}$

$\frac{11-3x}{x^2+2x+3} = \frac{11-3x}{(x-1)(x+3)} \Rightarrow \frac{A}{(x-1)} + \frac{B}{(x+3)}$

$\frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$

$A(x+3)+B(x-1) = 11-3x$

i $x = -3$

$B(-4) = 11-3(-3) = 11+9$

$B = -5$

i $x = 1$

$A(4) = 8$

$A = 2$

$\therefore \int \frac{2}{(x-1)}dx + \int \frac{-5}{(x+3)}dx = \int \frac{11-3x}{x^2+2x-3}dx$

$2\ln(x-1) - 5\ln(x+3) + c$

2) $\int \frac{4x-16}{x^2-2x-3}dx \quad = \frac{4x-16}{(x+1)(x-3)}$

$\frac{A}{x+1} + \frac{B}{x-3}$

$A(x-3)+B(x+1) = 4x-16$

$Ax - 3A + Bx + B = 4x - 16$

$x(A+B) - 3A + B = 4x - 16$

Hence;

$\begin{cases} A+B = 4 \\ -3A+B = -16 \end{cases}$

Find B

$A = 4-B$

$\therefore -3(4-B)+B = -16$

Find A

$B = 4-A$

$-3A+4-A = -1$

$\frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$

$A(x+3)+B(x-1)=11-3x$

$x=-3$

$B(-4)=11-3(-3)=11+9$

$B=-5$

$x=1$

$A(4)=8$

$A=2$

$\therefore \int \frac{2}{(x-1)}dx+\int \frac{-5}{(x+3)}dx=\int \frac{11-3x}{x^2+2x-3}dx$

$2\ln(x-1)-5\ln(x+3)+c$

2) $\int \frac{4x-16}{x^2-2x-3}dx$ $=\frac{4x-16}{(x+1)(x-3)}$

$\frac{A}{x+1}+\frac{B}{x-3}$

$A(x-3)+B(x+1)=4x-16$

$Ax-3A+Bx+B=4x-16$

$x(A+B)-3A+B=4x-16$

Hence;

$\begin{cases} A+B=4 \\ -3A+B=-16 \end{cases}$

Find B

$A=4-B$

$\therefore -3(4-B)+B=-16$

$-12+3B+B=-16$

$4B=-4$

$B=-1$

Find A

$B=4-A$

$-3A+4-A=$

$-4A=-2$

$A=5$

$\therefore =5\ln|x+1|-\ln|x-3|+C$

(3) $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$ $\quad \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$

$A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)=2x^2-9x-35$

$A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2)=2x^2-9x-35$

$Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C=2x^2-9x-35$

$Ax^2-Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C=$

$2x^2-9x-35$

$x^2(A+B+C)+x(A+4B-C)-6A+3B-2C=2x^2-9x$

$A+B+C=2$ ——— ①

$A+4B+(-C)=-9$ ——— ②

$-6A+3B-2C=-35$ ——— ③

$A=2-B-C$ ——— ④

$\therefore 2-B-C+4B-C=-9$

$-2C+3B=-11$ ——— ⑤

④ into ③

$-6(2-B-C)+3B-2C=-35$

$-12+6B+6C+3B-2C=-35$

$4C+9B=-23$ ——— ⑥

$-2C+3B=-11 \quad \times 3$

$4C+9B=-23 \times 1$

$-6C+9B=-33$

$4C+9B=-23$

$+10C=+10$

$C=1$

Using eq ⑥

$4C+9B=-23$

$4(1)+9B=-23$

$4+9B=-23$

$9B=-27$

$B=-3$

Using eq (4)

$A = 2 - B - C$

$A = 2 - (-3) - 1$

$A = 2 + 3 - 1$

$A = 4$

Then:

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$= \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$$

$$= 4\int \frac{dx}{x+1} - 3\int \frac{dx}{x-2} + 1\int \frac{dx}{x+3}$$

~~$= 4\ln|x+1| - 3\ln$~~

$= 4\ln|x+1| - 3\ln 1 + \ln|x+3|$