```
commandwindow
clear
clc
close all
syms t m(t) eqn sol(t)
%wt is amount of water in tank,so is the amount of solution tht leaves per minute
wt=1200
so=30
%This is the condition for the equation,that is at initial time m is 150lb
cond=m(0)==150;

%min is input rate of salt to the system,mout is output rate of salt to the
%system
min=50*(1+sin(t))
mout=(so/wt)*m

eqn=diff(m,t)==min-(mout)
sol(t)=dsolve(eqn,cond)
fprintf('Therefore the differential equation for studying the dynamics of the amount of salt in the tank is %s and the solution %s \n',char(eqn),char(sol(t)));

%7.5 hrs is converted to min,multiplied by 60
t=0:0.5:(7.5*60)
M=subs(sol,t);
plot(t,M)
title('Figure 1: Plot of Dynamic Response of amount of salt present in tank')
xlabel('Time(min)')
ylabel('Amount of Salt in Tank(lb)')
grid on
grid minor
```

Applying balance law.

$$\begin{bmatrix}\text{Accumulation rate}\\ \text{of salt within a system}\end{bmatrix} = \begin{bmatrix}\text{Input rate of}\\ \text{salt in the system}\end{bmatrix} - \begin{bmatrix}\text{Output rate of}\\ \text{salt from system}\end{bmatrix}$$

i.e $\frac{dm}{dt} = m_{in} - m_{out}$

$$\frac{dm}{dt} = \left[50(1+\sin t)\right] - \left[\frac{30}{1200} \times m\right]$$

$$\frac{dm}{dt} = 50(1+\sin t) - \frac{m}{40} \ldots$$

rewritten as

$$\frac{dm}{dt} + \frac{m}{40} = 50(1+\sin t)$$

using integrating factor

$$\int P\,dt = \int \frac{1}{40} = \frac{t}{40} \quad \therefore \text{ I.f.} = e^{t/40}$$

recall

$$m \times \text{I.f.} = \int Q \times \text{I.f.}$$

$$m \cdot e^{t/40} = \int 50(1+\sin t) \times e^{t/40}$$

$$= 50\int (1+\sin t)e^{t/40}$$

$$= 50\left(\int e^{t/40} + \int e^{t/40}\sin t\right.$$

$$= 50\left(40e^{t/40} + \int e^{t/40}\sin t \cdot dt\right. \quad \text{--X}$$

Solving $\int e^{t/40}\sin t \cdot dt$

integration by Parts ; $\int u\,dv = uv - \int v\,du$

$u = \sin t$, $du = \cos t$; $dv = e^{t/40}$ $v = 40e^{t/40}$

$$\therefore \int e^{t/40}\sin t \cdot dt = 40e^{t/40} \times \sin t - \int 40e^{t/40} \times \cos t \cdot dt \quad \text{--XX}$$

Solving $\int 40e^{t/40} \times \cos t \cdot dt$; integration by Parts

$u = \cos t$ $du = -\sin t$; $dv = 40e^{t/40}$ $v = 1600e^{t/40}$

$$\therefore \int 40e^{t/40}\cos t \cdot dt = 1600e^{t/40}\cos t - \int -1600e^{t/40}\sin t$$

$$= 1600e^{t/40}\cos t + 1600\int e^{t/40}\sin t \quad \text{--XXX}$$

Substituting xxx in X due to the reoccurence of $\int e^{t/40} \sin t$

$$\int e^{t/40} \sin t \cdot dt = 40e^{t/40} \sin t - (1600e^{t/40} \cos t + 1600 \cdot \int e^{t/40} \sin t$$

$$= 40e^{t/40} \sin t - 1600e^{t/40} \cos t - 1600 \int e^{t/40} \sin t$$

Solving for $\int e^{t/40} \sin t$

$$\int e^{t/40} \sin t \, dt + 1600 \int e^{t/40} \sin t \cdot dt = 40e^{t/40} \sin t - 1600e^{t/40} \cos t$$

$$1601 \int e^{t/40} \sin t \cdot dt = 40e^{t/40} \sin t - 1600e^{t/40} \cos t$$

$$\int e^{t/40} \sin t \cdot dt = \frac{40e^{t/40} \sin t - 1600e^{t/40} \cos t}{1601} \quad \text{-- XXXX}$$

Substituting XXXX in X

$$m \cdot e^{t/40} = 50\left(40e^{t/40} + \frac{40e^{t/40} \sin t - 1600e^{t/40} \cos t}{1601}\right)$$

$$m \cdot e^{t/40} = 2000e^{t/40} + \frac{2000e^{t/40} \sin t - 80000e^{t/40} \cos t}{1601} + C$$

dividing both sides by $e^{t/40}$

$$m = 2000 + \frac{2000 \sin t - 80000 \cos t}{1601} + Ce^{-t/40}$$

$C = m_0$; when $t = 0$ $m = 150$

$$150 = 2000 + \frac{2000 \sin(0) - 80000 \cos(0)}{1601} + Ce^{-0/40}$$

$$150 = 2000 + \frac{-80000}{1601} + m_0$$

$$150 = 1950 + m_0$$

$$m_0 = 150 - 1950$$

$$m_0 = -1800$$

$\therefore$ Eqn for $m = 2000 + \dfrac{2000 \sin t - 80000 \cos t}{1601} - 1800e^{-t/40}$

MATLAB R2018a

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Command Window

```
wt =

        1200


so =

    30


min =

50*sin(t) + 50


mout(t) =

m(t)/40


eqn(t) =

diff(m(t), t) == 50*sin(t) - m(t)/40 + 50


sol(t) =

2000 - (2000*1601^(1/2)*cos(t + atan(1/40)))/1601 - (2881850*exp(-t/40))/1601
```

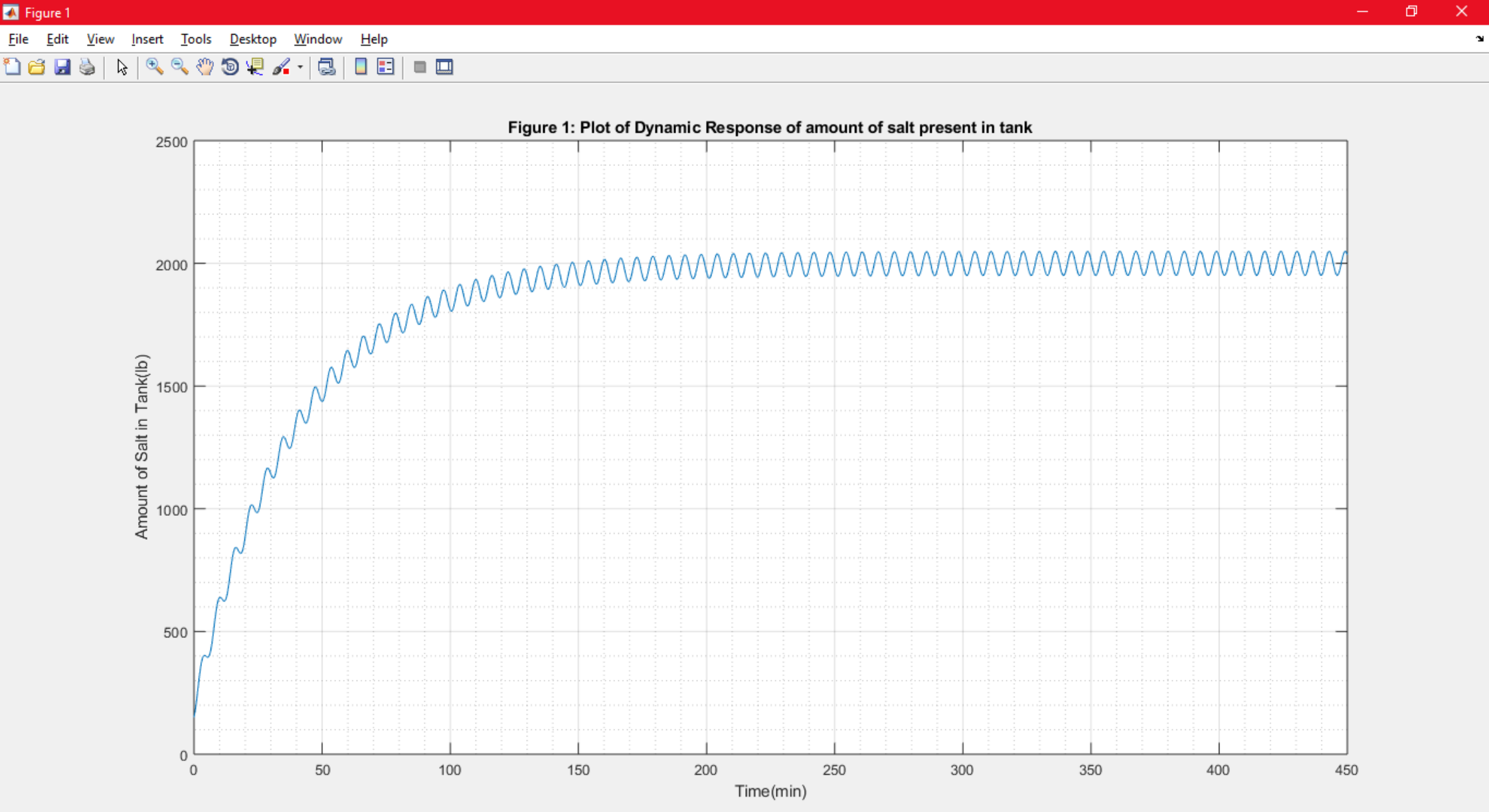
Figure 1
File Edit View Insert Tools Desktop Window Help
Figure 1: Plot of Dynamic Response of amount of salt present in tank
Amount of Salt in Tank(lb)
Time(min)
0
500
1000
1500
2000
2500
0
50
100
150
200
250
300
350
400
450

```
commandwindow
clear
clc
close all
syms t y ym
y=(50/0.05)+((50/1.0025)*sin(t))+(((50*0.05)/1.0025))*cos(t)-(802.49*exp(-0.05*t))
ym=1000-(800*exp(-0.05*t))

figure(2)
hold all
for t= 0:1:500
    if rem(t,2)==1
        V=subs(y,t)
        else
        V=subs(ym,t)
    end
    plot(t,V,'-*r')
    v=double(V)
end

title('Figure 2:Dynamic Model Of System')
xlabel('Time(min)')
ylabel('Response(Litre)')
grid on
grid minor

column={'t(min)','V(litre)'};
xlswrite('odevbesdata',[t(:),v(:)],'Veriler','A2');
xlswrite('odevbesdata',column, 'Veriler','A1');
```

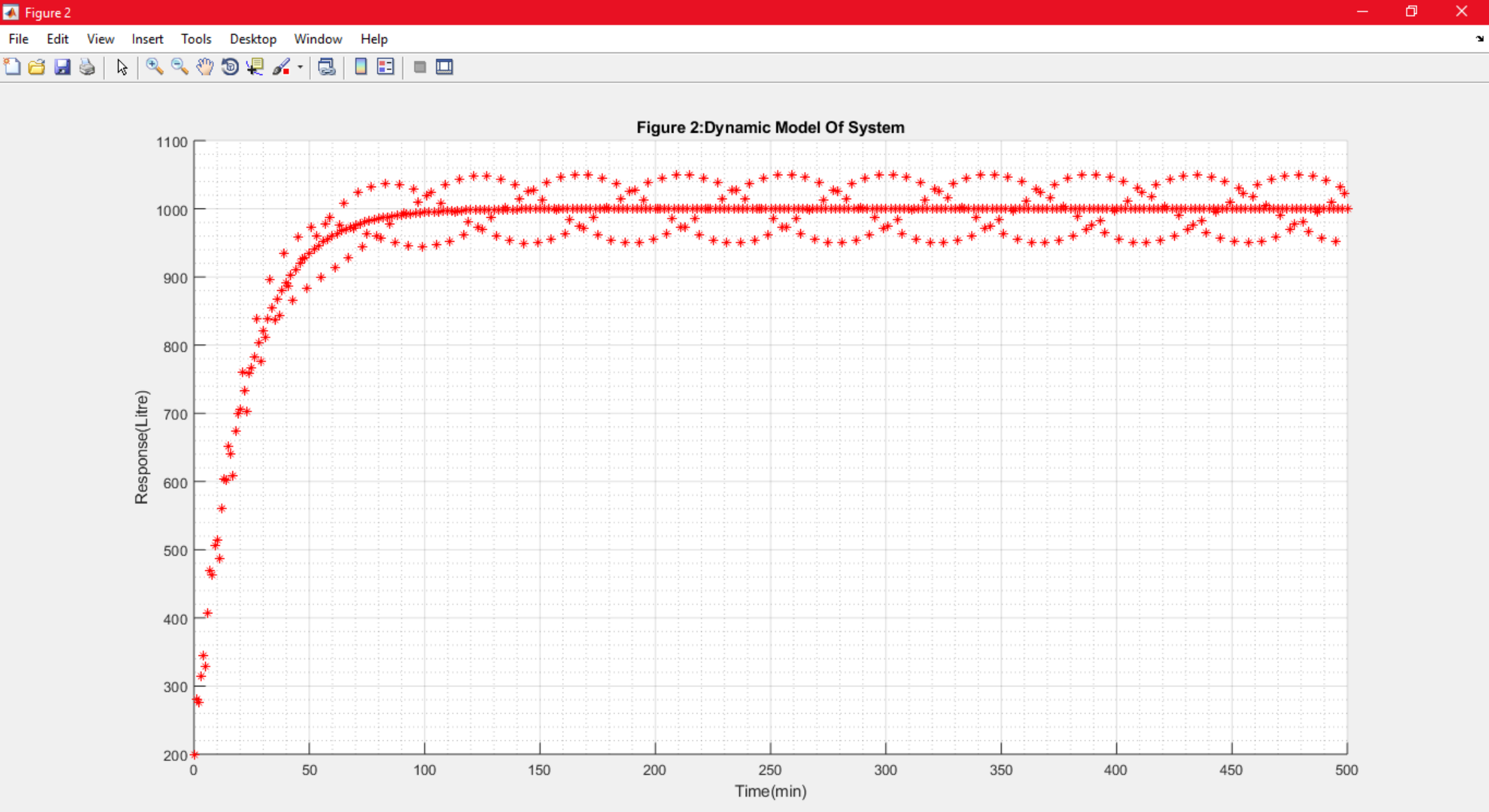
Figure 2
File Edit View Insert Tools Desktop Window Help
Figure 2:Dynamic Model Of System
Response(Litre)
Time(min)
1100
1000
900
800
700
600
500
400
300
200
0
50
100
150
200
250
300
350
400
450
500

odevbesdata [Compatibility Mode] - Excel (Product Activation Failed)

FILE HOME INSERT PAGE LAYOUT FORMULAS DATA REVIEW VIEW

A1 t(min)

t(min)	V(litre)
500	1000
1	239.0165
2	276.1301
3	311.4336
4	345.0154
5	376.9594
6	407.3454
7	436.2495
8	463.744
9	489.8975
10	514.7755
11	538.4402
12	560.9507
13	582.3634
14	602.7318
15	622.1068
16	640.5368
17	658.0681
18	674.7443
19	690.6072
20	705.6964
21	720.0498
22	733.7031
23	746.6906
24	759.0446
25	770.7962
26	781.9746
27	792.6078
28	802.7224

Sheet1 Veriler

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