

$$1 \quad \begin{aligned} \vec{M} &= p\vec{i} - 6\vec{j} - 3\vec{k} \\ \vec{N} &= 4\vec{i} + 3\vec{j} - \vec{k} \\ \vec{O} &= \vec{i} - 3\vec{j} + 2\vec{k} \end{aligned}$$

$$1 \quad \vec{M} \cdot \vec{N} = 0 \\ (p\vec{i} - 6\vec{j} - 3\vec{k}) \cdot (4\vec{i} + 3\vec{j} - \vec{k}) = 0 \\ +p - 18 + 3k = 0 \\ 4p - 18 + 3 = 0 \\ 4p = 15 \\ p = 15/4 = 3.75$$

$$b) \quad \vec{M} \cdot |\vec{N} \times \vec{O}| = 0$$

$$|\vec{N} \times \vec{O}| = \begin{vmatrix} 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} \\ = \vec{i}(6 - (-3)) - \vec{j}(8 - (-1)) + \vec{k}(-12 - 3) \\ |\vec{N} \times \vec{O}| = 3\vec{i} - 9\vec{j} - 15\vec{k}$$

$$\vec{M} \cdot |\vec{N} \times \vec{O}| \\ = p\vec{i} - 6\vec{j} - 3\vec{k} \cdot (3\vec{i} - 9\vec{j} - 15\vec{k}) \\ = 3p - 48 + 45 \\ 3p - 48 + 45 = 0 \\ 3p = 3 \\ p = 1$$

$$2 \quad (3\vec{i} + 2\vec{j} + 5\vec{k}) + (2\vec{i} - \vec{j} + \vec{k}) + (5\vec{i} + 2\vec{j} - 3\vec{k}) \\ = 10\vec{i} + 3\vec{j} + 8\vec{k} \\ |\vec{a}| = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{173}$$

$$\cos \alpha = \frac{9}{10} = 0.9000$$

$$\cos \beta = \frac{3}{10} = 0.3000$$

$$\cos \gamma = \frac{8}{10} = 0.8000$$

Unit vector =  $\frac{10\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}}{\sqrt{10^2 + 3^2 + 8^2}}$

$$\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} + (u+2)\mathbf{k}$$

$$\mathbf{V} = 2u\mathbf{i} + (-3u)\mathbf{j} + (u+2)\mathbf{k}$$

$$\begin{aligned} (\mathbf{F} \times \mathbf{V}) &= \begin{vmatrix} 2 & 3 & u+2 \\ 2u & -3u & u+2 \end{vmatrix} \\ &= \mathbf{i} [u^2(u+2) - 3u(u+2)] - \mathbf{j} [2u(u+2) - (u+2)2u] + \mathbf{k} [2u - 2u] \\ &= (u^3 - u^2 + 2u)\mathbf{i} - (2u^2 + 2u - 2u^2)\mathbf{j} - (2u - 2u)\mathbf{k} \\ &= (u^3 - u^2 + 2u)\mathbf{i} - (2u - 2u)\mathbf{j} - (2u - 2u)\mathbf{k} \end{aligned}$$

$$= \int (u^3 - u^2 + 3u) - \int (3u - 6 - 2u^2) i - \int (1 - 2u^2) k$$

$$= \frac{u^4}{4} - \frac{u^3}{3} + 3u^2 i - \left( \frac{-3u^2 + u^3}{2} \right) i - \left( \frac{9u^3 - u^4}{2} \right) k$$

$$= \left. \frac{u^4}{4} - \frac{u^3}{3} + 3u^2 i + \frac{3u^2 - u^3}{2} i - \frac{9u^3 - u^4}{2} k \right|_0^1$$

$$= \left. \frac{\text{For } i}{\left[ \frac{1^4}{4} - \frac{1^3}{3} + 3 \times 1^2 - \left( \frac{3 \times 0^2 - 0^3}{2} + 3 \times 0^2 \right) \right]} \right|_0^1$$

$$= \frac{35}{12} i +$$

$$\left[ \frac{\text{For } j}{\frac{3 \times 1^2 + 1^3}{2} - \left( \frac{3 \times 0^2 + 0^3}{2} \right)} \right] j$$

$$= 2j$$

$$\left[ \frac{\text{For } k}{\frac{9 \times 1^3 - 1^4}{2} - \frac{9 \times 0^3 - 0^4}{2}} \right] k$$

$$= 4k$$

$$\therefore \int_C \mathbf{F} \cdot d\mathbf{u} = \frac{35}{12} i + 2j + 4k$$