

NAME: IJUWE ALEXANDER I

MAT. NO.: 19/ENG04/061

DEPT: ELECTRICAL ENGINEERING

ENGINEERING MATHEMATICS ASSIGNMENT V

Question 1

Accumulation rate of salt = input rate of salt - output rate of salt
within a system into the system from the system

Denoting the amount of salt present in the tank at any time t as y , its time rate of change is given as

$$\frac{dy}{dt} = Y_{in} - Y_{out}$$

Since 50 gal of brine enter the tank per minute and one gallon contains $(1 + \sin t)$ lb of salt;

a) at $t=1$; $(1 + \sin t) = (1 + \sin(1)) = 1.02$ lb of salt

Therefore it means that the amount of salt entering the tank is ~~1~~ min

$$Y_{in} = 50 \text{ gal/min} \times 1.02 \text{ lb/gal} = 51 \text{ lb/min}$$

The tank contains 1200 gal of water with the dissolved salt, $\frac{1}{30}$ 30 gallons of the solution leaves the tank per minute. That is $\frac{30 \text{ gal}}{1200 \text{ gal}}$

$0.025 = 2.5\%$ of the content of the tank, if that is the case, 2.5% of the salt present in the tank will also leave the tank per minute. In other words

$$Y_{out} = 2.5\% \text{ of } y$$

a) Therefore: $\frac{dy}{dt} \frac{\text{lb}}{\text{min}} = \frac{51 \text{ lb}}{\text{min}} - 2.5\% \text{ of } y \text{ lb/min}$

b) $\frac{dy}{dt} = 51 - 0.025y$; $\frac{dy}{dt} = -0.025y + 51$

$$\frac{dy}{dt} = -0.025 \left[\frac{-0.025y}{-0.025} + \frac{51}{-0.025} \right]; \frac{dy}{dt} = -0.025(y - 2040)$$

$$\frac{dy}{dt} = -0.025(y - 2040) \quad \frac{dy}{y - 2040} = -0.025 dt$$

$$\int \frac{dy}{(y-2040)} = \int -0.025 dt$$

$$\int \frac{dy}{(y-2040)} = -0.025 \int dt ; \ln(y-2040) = -0.025t + C;$$

$$y - 2040 = e^{-0.025t + C}$$

$$y - 2040 = e^{-0.025t} e^C$$

$$y - 2040 = e^{-0.025t} y_0$$

$$y = y_0 e^{-0.025t} + 2040, \text{ Given that when } t=0 \text{ (Initially)}$$

$$y = 15066$$

$$150 = y_0 e^{-0.025(0)} + 2040; 150 - 2040 = y_0 \times 1$$

$$y_0 = -1890$$

Therefore

$$y = 2040 - 1890e^{-0.025t}$$