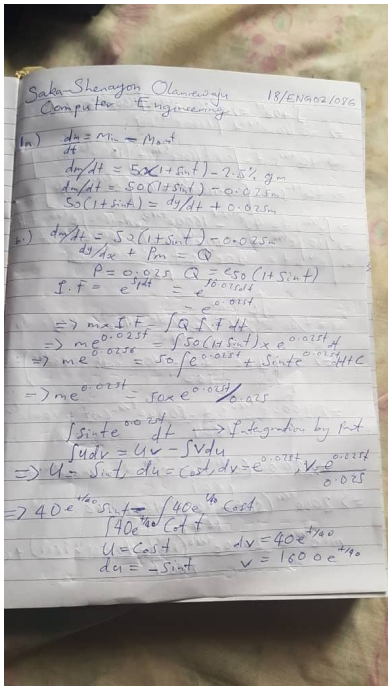


Saka-Shenayon Olamiwaju 18/ENG02/1096
Computer Engineering

(a) $\frac{dm}{dt} = m_{in} - m_{out}$

$\frac{dy}{dt} = 5x[1+\sin t] - 2.5\% \, g_m$

$\frac{dm}{dt} = 50(1+\sin t) - 0.025m$

$50(1+\sin t) = \frac{dy}{dt} + 0.025m$

(b) $\frac{dy}{dt} = 50(1+\sin t) - 0.025m$

$\frac{dy}{dx} + P_m = Q$

$P = 0.025 \quad Q = e^{50}(1+\sin t)$

$I.f = e^{\int P dt} = e^{0.025t}$

$\Rightarrow m \times I.f = \int Q \, I.f \, dt$

$\Rightarrow m e^{0.025t} = \int 50(1+\sin t) \times e^{0.025t} dt$

$\Rightarrow m e^{0.025t} = 50 \int e^{0.025t} + \sin t \, e^{0.025t} + C$

$\Rightarrow m e^{0.025t} = 50 \times e^{0.025t} / 0.025$

$\int \sin t \, e^{0.025t} dt \rightarrow$ Integration by Part

$\int u dv = uv - \int v du$

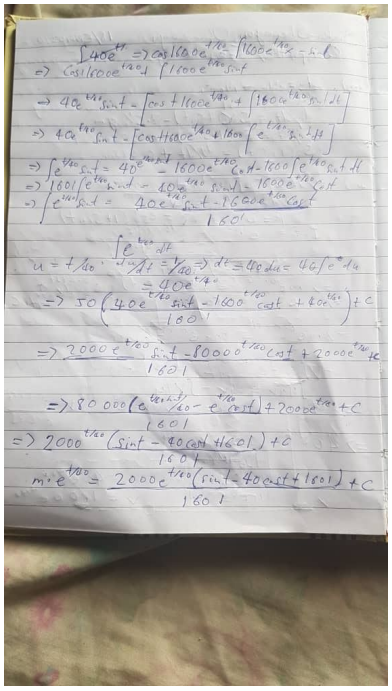
$\Rightarrow u = \sin t, \; du = \cos t, \; dv = e^{0.025t}, \; v = \frac{e^{0.025t}}{0.025}$

$\Rightarrow 40 e^{t/40} \sin t = \int 40 e^{t/40} \cos t$

$\int 40 e^{t/40} \cos t$

$u = \cos t \qquad dv = 40 e^{t/40}$

$du = -\sin t \qquad v = 1600 e^{t/40}$



$\int 40 e^{t/40} \Rightarrow \cos t 1600 e^{t/40} - \int 1600 e^{t/40} \times -\sin t$

$\Rightarrow \cos t 1600 e^{t/40} + \int 1600 e^{t/40} \sin t$

$\Rightarrow 40 e^{t/40} \sin t = \left[\cos t 1600 e^{t/40} + \int 1600 e^{t/40} \sin t\right]$

$\Rightarrow 40 e^{t/40} \sin t - \left[\cos t 1600 e^{t/40} + 1600 \int e^{t/40} \sin t\right]$

$\Rightarrow \int e^{t/40} \sin t = 40 e^{t/40} \sin t - 1600 e^{t/40} \cos t + \int 1600 e^{t/40} \sin t$

$\Rightarrow 1601 \int e^{t/40} \sin t = 40 e^{t/40} \sin t - 1600 e^{t/40} \cos t$

$\Rightarrow \int e^{t/40} \sin t = \frac{40 e^{t/40} \sin t - 1600 e^{t/40} \cos t}{1601}$

$\int e^{t/40} \sin t$

$u = t/40 \Rightarrow \frac{du}{dt} = \frac{1}{40} \Rightarrow dt = 40 du = 40 \int e^u du$

$= 40 e^{t/40}$

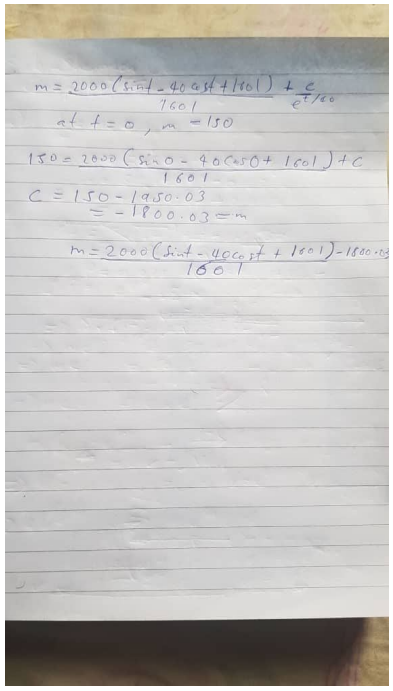
$\Rightarrow 50 \left(\frac{40 e^{t/40} \sin t - 1600 e^{t/40} \cos t - 1600 e^{t/40}}{1601}\right) + C$

$\Rightarrow \frac{2000 e^{t/40} \sin t - 80000 e^{t/40} \cos t + 2000 e^{t/40}}{1601} + C$

$\Rightarrow \frac{80000 \left(e^{t/40} \sin t - e^{t/40} \cos t\right)}{1601} + 2000 e^{t/40} + C$

$\Rightarrow \frac{2000 e^{t/40} (\sin t - 40 \cos t + 1601)}{1601} + C$

$m \cdot e^{t/40} = \frac{2000 e^{t/40} (\sin t - 40 \cos t + 1601)}{1601} + C$



$m = \frac{2000(\sin t - 40 \cos t + 1601)}{1601} + \frac{C}{e^{t/40}}$

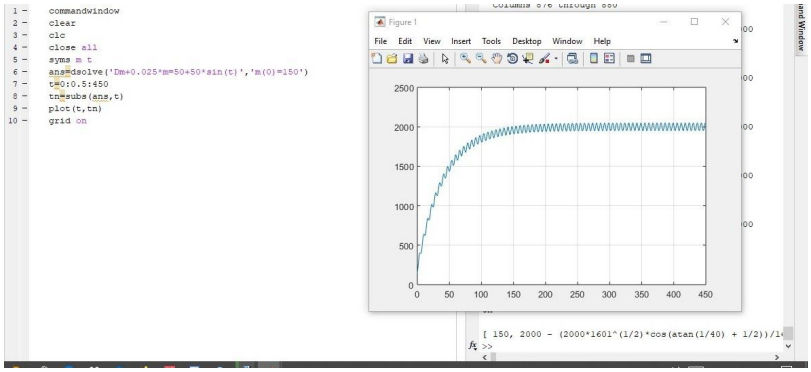
at $t = 0, \; m = 150$

$150 = \frac{2000(\sin 0 - 40 \cos 0 + 1601)}{1601} + C$

$C = 150 - 1950.03$

$= -1800.03 = m$

$m = \frac{2000(\sin t - 40 \cos t + 1601) - 1800.03}{1601}$



```
commandwindow
clear
clc
close all
syms m t
ans=dsolve('Dm+0.025*m=50+50*sin(t)','m(0)=150')
t=0:0.5:450
tn=subs(ans,t)
plot(t,tn)
grid on
```



```
[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1
fx >>
```