

MEZE HANNAH CHIBUEZE

18/ENG05/031

MECHATRONICS ENGINEERING

NUMBER 1:

me24 HW#11 CH#10  
 15/04/2021  
 MECHATRONICS ENGINEERING

(a) Applying balance law,  
 $\frac{dy}{dt} = Y_{in} - Y_{out}$

$$Y_{in} = 50 \frac{\text{gal}}{\text{min}} \times (1 + \sin t) \frac{\text{lb}}{\text{gal}} = 50(1 + \sin t) \frac{\text{lb}}{\text{min}}$$

$30 = 0.025 = 2.5\%$  of the salt present in the tank will leave the tank per minute.

$$Y_{out} = 2.5\% \text{ of } y$$

$$\frac{dy}{dt} = 50(1 + \sin t) - 0.025y$$

(b)  $\frac{dy}{dt} + 0.025y = 50(1 + \sin t)$   
 $\frac{dy}{dt} + Py = Q$

$$P = 0.025, \quad Q = 50(1 + \sin t)$$

$$I.F = e^{\int P dt}, \quad \int P dt = 0.025t, \quad I.F = e^{0.025t}$$

$$y \cdot I.F = \int Q \cdot I.F$$

$$y e^{0.025t} = \int 50(1 + \sin t) e^{0.025t} dt$$

$$y e^{0.025t} = 50 \left( \int e^{0.025t} + e^{0.025t} \sin t \cdot dt \right)$$

$$y e^{0.025t} = 50 \left( \frac{e^{0.025t}}{0.025} \right) + \int e^{0.025t} \sin t dt$$

$\int e^{0.025t} \sin t dt$ , solving by integrating by part,  
 $\int u dv = u v - \int u' (v) dx$

$$u = e^{0.025t} : \frac{du}{dt} = 0.025 e^{0.025t}$$

$$v = \sin t : \int v dt = \int \sin t dt = -\cos t$$

$$\int e^{0.025t} \sin t dt = -e^{0.025t} \cos t + 0.025 e^{0.025t} \cos t + C$$

isn't

Integrating by part again,  
 $= -e^{0.025t} \cos t + 0.025 \left( e^{0.025t} \sin t - 0.025 \int e^{0.025t} \sin t \right)$

$$P = e^{-0.025t} \text{ let } p = \int e^{0.025t} \sin t$$

$$P = -e^{-0.025t} \cos t + 0.025 \int e^{0.025t} \sin t - 0.025 P$$

$$P + 0.000625 P = -e^{-0.025t} \cos t + 0.025 \int e^{0.025t} \sin t - 0.000625 P$$

$$1.000625 P = -e^{-0.025t} \cos t + 0.025 \int e^{0.025t} \sin t$$

$$P = -e^{-0.025t} (\cos t - 0.025 \sin t) + C$$

$$y = 2000$$

$$y e^{0.025t} = 50 \left( \frac{e^{0.025t}}{0.025} + e^{0.025t} (\cos t - 0.025 \sin t) + C \right)$$

$$y e^{0.025t} = 2000 e^{0.025t} + 49.97 e^{0.025t} (\cos t - 0.025 \sin t) + 50 y_0$$

$$y = 2000 - 49.97 (\cos t - 0.025 \sin t) + 50 y_0 e^{-0.025t}$$

Given that when  $t=0$  (initially),  $y=150$  lb

$$150 = 2000 - 49.97(1-0) + 50 y_0$$

$$y_0 = -36$$

$$y = 2000 - 49.97(\cos t - 0.025 \sin t) - 1800 e^{-0.025t}$$

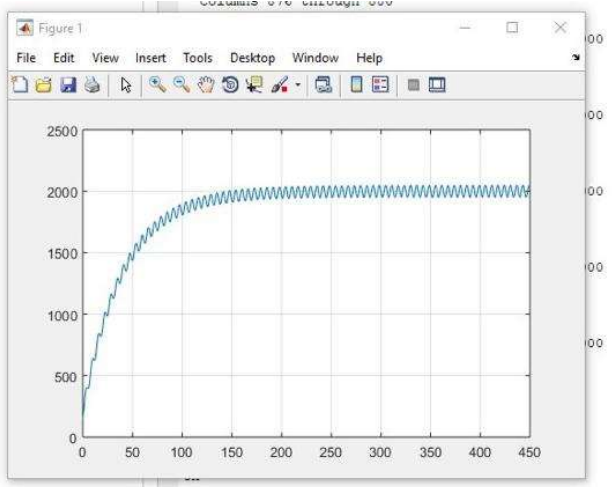
Recall; amount of salt at time  $t = m$

$$m = 2000 - 49.97(\cos t - 0.025 \sin t) - \frac{1800}{e^{0.025t}}$$

```

1  commandwindow
2  clear
3  clc
4  close all
5  syms m t
6  ans=dsolve('Dm+0.025*m=50+50*sin(t)', 'm(0)=150')
7  t=0:0.5:450
8  tn=subs(ans,t)
9  plot(t,tn)
10 grid on

```



$$[ 150, 2000 - (2000 \cdot 1601^{1/2}) \cos(\text{atan}(1/40) + 1/2) ] / 1601$$

## NUMBER 2:

