

$$1) \frac{dm}{dt} = -\frac{m}{40} + 50(1 + \sin(t))$$

$$\frac{dm}{dt} + \frac{m}{40} = 50(1 + \sin(t))$$

using integrating factor $e^{\int P dt}$

$$\int P dt = \int \frac{1}{40} = \frac{t}{40} \therefore e^{\int P dt} = e^{t/40}$$

Recall

$$\text{Int. IF} = \int Q \text{ IF}$$

$$m \cdot e^{t/40} = \int (50(1 + \sin(t)) \cdot e^{t/40}$$

$$m \cdot e^{t/40} = 50 \int (1 + \sin(t)) \cdot e^{t/40}$$

$$m \cdot e^{t/40} = 50 \int e^{t/40} + \sin(t) \cdot e^{t/40}$$

$$= 50 \left(\int e^{t/40} dt + \int e^{t/40} \sin(t) dt \right)$$

$$m \cdot e^{t/40} = 50 \left(40e^{t/40} + \int e^{t/40} \sin(t) dt \right) \quad \text{eqn 1}$$

solving $\int e^{t/40} \sin(t) dt$

making use of integration by parts $\int u dv = uv - \int v du$

$$u = \sin(t) \quad dv = e^{t/40}, \quad du = \cos(t) \quad v = 40e^{t/40} \quad \text{eqn 2}$$

$$\int e^{t/40} \sin(t) dt = 40e^{t/40} \sin(t) - \int 40e^{t/40} \cos(t) dt$$

Solving $\int 40e^{t/40} \cos(t) dt$

using integration by parts $\int u dv = uv - \int v du$

$$u = \cos(t) \quad dv = 40e^{t/40} \cos(t) = \int -1600e^{t/40} \sin(t)$$

$$= 1600e^{t/40} \cos(t) + 1600 \int e^{t/40} \sin(t) \quad \text{eqn 3}$$

substituting 3 into 2 due to re occurrence of $e^{t/40} \sin(t)$

$$\int e^{t/40} \sin(t) dt = 40e^{t/40} \sin(t) - (1600e^{t/40} \cos(t) + 1600 \int e^{t/40} \sin(t) dt)$$

$$= 40e^{t/40} \sin(t) - 1600e^{t/40} \cos(t) - 1600 \int e^{t/40} \sin(t) dt$$

making $\int e^{t/40} \sin(t) dt$ the subject, simplifying

$$\int e^{t/40} \sin(t) dt + 1600 \int e^{t/40} \sin(t) dt = 40e^{t/40} \sin(t) - 1600e^{t/40} \cos(t)$$

$$1601 \int e^{t/40} \sin(t) dt = 40e^{t/40} \sin(t) - 1600e^{t/40} \cos(t)$$

$$\int e^{t/40} \sin(t) dt = \frac{40e^{t/40} \sin(t) - 1600e^{t/40} \cos(t)}{1601} \quad \text{eqn 4}$$



Substituting eqn (5) in (4)

$$m \cdot e^{t/40} = 50 \left(40e^{t/40} + \frac{40e^{t/40} \sin(t) - 1600e^{t/40} \cos(t)}{1601} \right)$$

$$m \cdot e^{t/40} = 2000e^{t/40} + \frac{2000e^{t/40} \sin(t) - 8000e^{t/40} \cos(t)}{1601} + C$$

Dividing both sides by $e^{t/40}$

$$m = 2000 + \frac{2000 \sin(t) - 8000 \cos(t)}{1601} + C e^{-t/40}$$

$C = m_0$, when $t = 0$ $m = 150$

$$150 = 2000 + \frac{2000 \sin(0) - 8000 \cos(0)}{1601} + m_0 e^{-0/40}$$

$$150 = 2000 + \left(-\frac{8000}{1601} \right) + m_0$$

$$150 = 1950 + m_0$$

$$m_0 = 150 - 1950$$

$$m_0 = -1800 = 1800$$

$$\therefore \text{Equation for } m = 2000 + \frac{2000 \sin(t) - 8000 \cos(t)}{1601} + 1800 e^{-t/40}$$

```
1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - syms m t
6 - M=dsolve('Dm + (0.025*m)=50*sin(t)+50', 'm(0)=150')
7 - t=0:0.5:450;
8 - Mn=subs(M,t)
9 - plot(t,Mn,'black')
10 - xlabel('Time (min)')
11 - ylabel(' SALT PRESENT (Ib)')
12 - grid on
13 - grid minor
14
15
```

Command Window

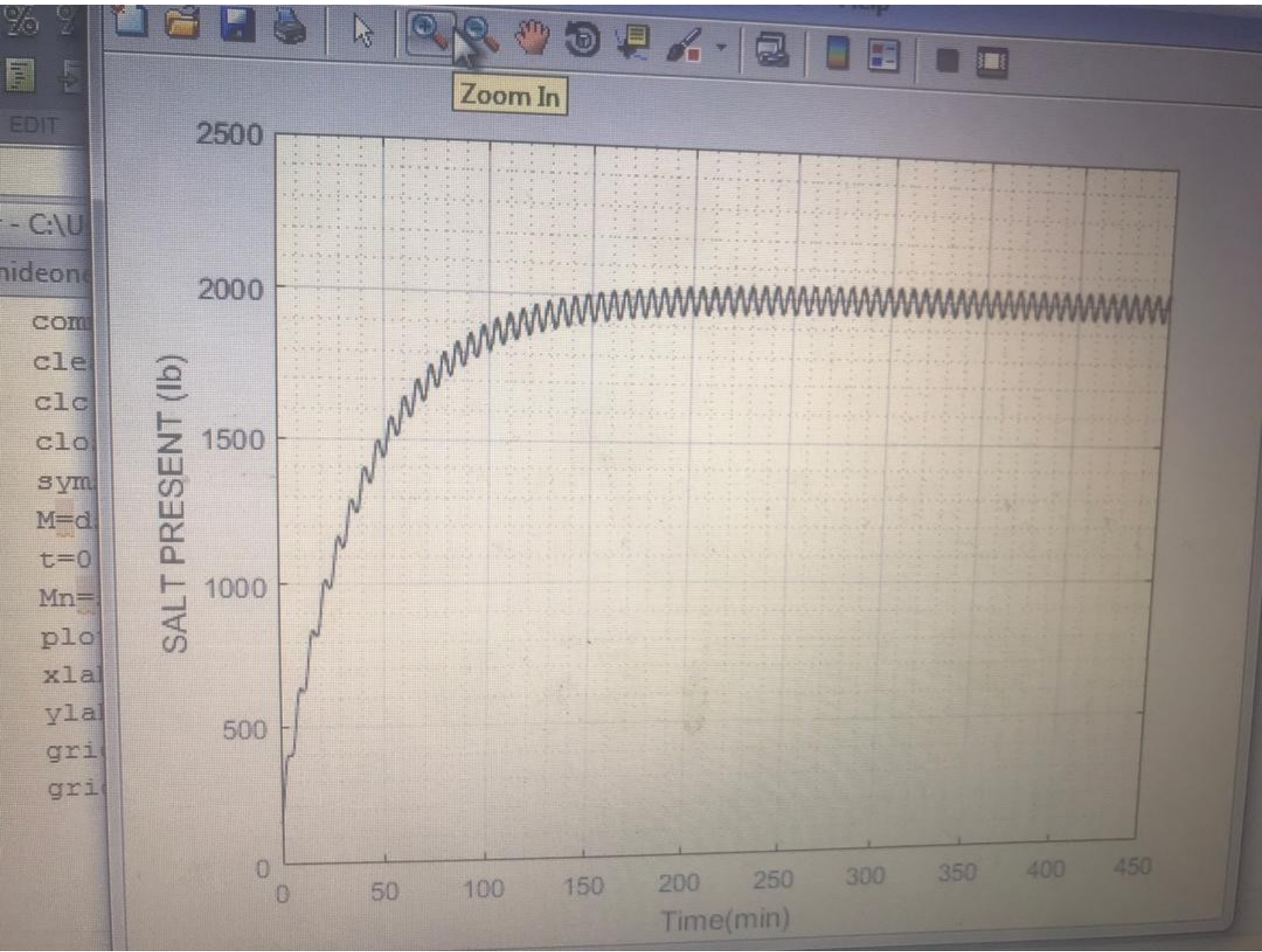
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```
2000 - (2000*1601^(1/2)*cos(t + atan(1/40)))/1601 - (2881850*exp(-t/40))/1601
```

Mn =

```
[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/
```

```
>> |
```



```
close all
5 =xlsread('Assignment 5','number 2')
6 syms t1 t2
7 y=(50/0.05)+(50*sin(t)/1.0025)+(50*0.05*cos(t)/1.0025)-(802.49*exp(-0.05*t))
8 ym=1000-(800*exp(-0.05*t))
9 ys=y(1:2:500,1)
10 ts=t(1:2:500,1)
11 yms=y(2:2:500,1)
12 tms=t(2:2:500,1)
13 plot(ys,ts,'blue')
14 plot(yms,tms,'red')
15 grid on
16 grid minor
17 legend('y','y_m')
18 xlabel('t (min)')
19 ylabel('V(liter)')
20
```

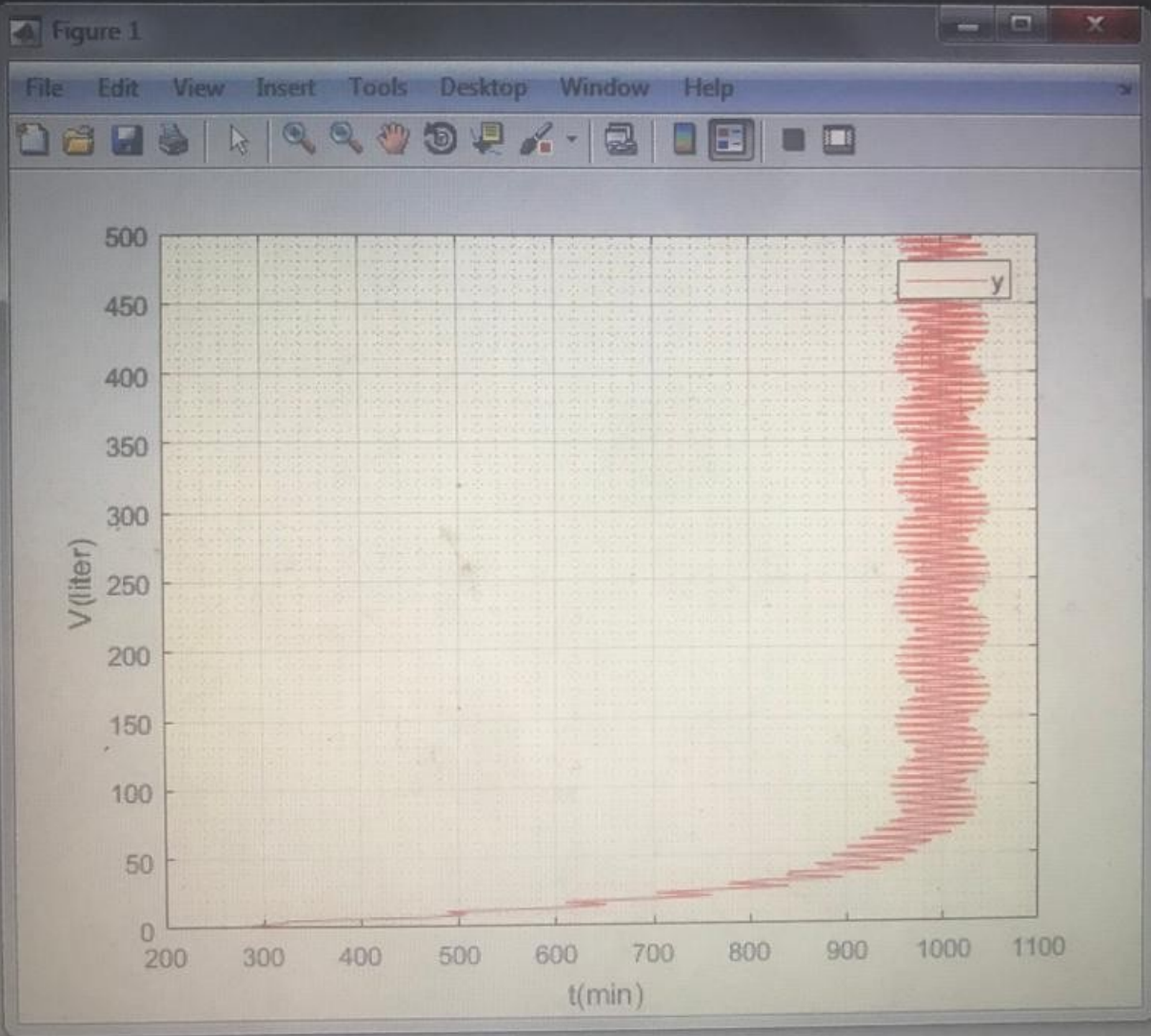
Command Window

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$$2000 - (2000 \cdot 1601^{(1/2)} \cdot \cos(t + \text{atan}(1/40)))/1601 - (2881850 \cdot \exp(-t/40))/1601$$

Mn =

$$[150, 2000 - (2000 \cdot 1601^{(1/2)} \cdot \cos(\text{atan}(1/40) + 1/2))/1601 - (2881850 \cdot \exp(-1/40))/1601,$$



05*t))

Command Window

