

18MHS01322.

ELECTRICAL ENGINEERING.

ENGINEERING MATH ASSIGNMENT 5

1a. Water amount = 1200

Salt amount = 150 lb

Input rate: 50 (water) lb in each gallon

Output rate: 30 gal

$$\frac{dy}{dx} = f_{in} - f_{out}; \quad f_{in} = \frac{50 \text{ gal} \times (\text{lb salt})}{1 \text{ in} \quad 1 \text{ gal}} = (50 + 50 \sin t) \text{ lb}$$

$$f_{out} = \frac{30}{1200} = 0.025 = 2.5\% \text{ of } y$$

$$\frac{dy}{dx} = 50 + 50 \sin t - 0.025 y \Rightarrow \text{Ordinary differential equation.}$$

$$b. \frac{dy}{dx} + 0.025y = (50 + 50 \sin t)$$

$$\text{For } \frac{dy}{dx} + py + Q = 0; \quad p = 0.025$$

$$p dx = 0.025 t$$

$$I = e^{\int p dx} = e^{0.025 t}$$

$$\therefore y I = \int Q \cdot I$$

$$y \cdot e^{0.025 t} = \int (50 + 50 \sin t) \cdot e^{0.025 t}$$

Integrating the R.H.S.

$$\Rightarrow 200 e^{0.025 t} + 50 e^{0.025 t} \sin t \text{ where } \int 50 e^{0.025 t} \sin t = P$$

$$\text{Let } u = \sin t \quad du = e^{0.025 t} dt$$

$$dy = \sin t dt \quad v = \frac{e^{0.025 t}}{0.025}$$

$$= \frac{50 \sin t \cdot e^{0.025 t}}{0.025} - \int \frac{e^{0.025 t}}{0.025} (10 \sin t) dt \text{ for } P$$

$$= 200 \sin t \cdot e^{0.025 t} - \int 2000 e^{0.025 t} \sin t$$

INTEGRATING $\int 2000 e^{0.025t} \cos t$

$$\Rightarrow 2000 \int e^{0.025t} \cos t$$

$$\text{Let } u = \cos t \quad du = -e^{0.025t} dt$$

$$du = \sin t \quad v = 40e^{0.025t}$$

From Sudo: $dv - du$

$$\Rightarrow 2000 (40e^{0.025t} \cos t) + 80,000 \int \sin t dt e^{0.025t}$$

$$\Rightarrow 80,000 e^{0.025t} \cos t + 80,000 \int \sin t dt e^{0.025t}$$

$$\Rightarrow 80,000 e^{0.025t} \cos t + 1600 \int \sin t dt + 160000 e^{0.025t} dt$$

Recall $\int \sin t dt = -\cos t$

$$= 80,000 e^{0.025t} \cos t + 1600 \int$$

From eq 2

$$[80,000 \sin t e^{0.025t} = 2000 (t) e^{0.025t} - (80,000 e^{0.025t} \cos t) + 1600 \int]$$

$$I = 20,000 \sin t e^{0.025t} - 80,000 e^{0.025t} \cos t - 1600 \int$$

$$1601 I = \exp(t/40) (2000 \sin t - 80,000 \cos t)$$

$$1601 I = \exp(t/40) \left(80,000 \left(\frac{\sin t}{40} - \cos t \right) \right)$$

$$I = \exp(t/40) \left(80,000 \left(\sin t/40 - \cos t \right) \right) \times \frac{1}{1601}$$

From $y' + P y = Q$

$$\exp(t/40) = 2000 \exp(t/40) + \left(\frac{\sin t}{40} \cos t - \cos t \right) 80,000 \exp(t/40)$$

$$y = 2000 + \left(\frac{\sin t}{40} - \cos t \right) 80,000 \times \frac{1}{1601}$$

$$y = 2000 + \frac{1}{1601} \left(80,000 \left(\frac{\sin t}{40} - \cos t \right) \right)$$

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1 commandwindow
2 clearvars
3 clc
4 close all
5
6 syms m(t) t
7 eqn = diff(m,t) == (50+50*sin(t))-0.025*
8 cond = m(0)==150;
9 mSol(t) = dsolve(eqn,m(0)==150)
10 t = 0:0.5:450;
11 plot(t,mSol(t))
12 xlabel('Time(hr)')
13 ylabel('Amount of substance(lb)')
14 grid on
15 grid minor
```

```
16 hold off  
17 grid on  
18 grid minor
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