

NAME : Dhaqo Tmunomibaka Samson

DEPARTMENT : MECHANICAL ENGINEERING

MATRIC NO: 18/ENAG05/013

1) Water amount = 1200 gal

Salt amount = 150 lb

Input rate = 50 gal/min; (1 + sin t) lb in each gallon

Output rate = 30 gal/min;

$$a) \frac{dy}{dt} = y_{in} - y_{out}; y_{in} = \frac{50 \text{ gal}}{1 \text{ min}} \times \frac{(1 + \sin t)}{1 \text{ gal}}$$
$$= (50 + 50 \sin t) \text{ lb/min}$$

$$y_{out} = \frac{30}{1200} = 0.025 = 2.5\%$$

$$\frac{dy}{dt} = (50 + 50 \sin t) - 0.025y \Rightarrow \text{ordinary differential equation}$$

$$b) \frac{dy}{dt} + 0.025y = (50 + 50 \sin t)$$

$$\text{From } \frac{dy}{dt} + Py = Q$$

$$\text{Let } P = 0.025$$

$$\int P dt = 0.025t$$

$$e^{\int P dt} = e^{0.025t} = IF$$

$$\therefore y \cdot IF = \int Q \cdot IF$$

$$y \cdot e^{0.025t} = \int (50 + 50 \sin t) e^{0.025t} \dots (3)$$

Integrating the R.H.S

$$\int 50 e^{0.025t} + 50 e^{0.025t} \sin t dt \Rightarrow$$

$$2000 e^{0.025t} + \int 50 e^{0.025t} \sin t dt$$

$$\text{for } \int 50 e^{0.025t} \sin t dt \Rightarrow 50 \int e^{0.025t} \sin t dt = I \dots (4)$$

$$= 50 \int e^{0.025t} \sin t dt = I$$

$$= u = \sin t$$

$$dv = e^{0.025t} dt$$

$$du = \cos t dt$$

$$v = \frac{e^{0.025t}}{0.025}$$

$$= 50 \left( \frac{\sin t e^{0.025t}}{0.025} - \int \frac{e^{0.025t}}{0.025} (\cos t) dt \right)$$

$$= 2000 \sin t e^{0.025t} - \int 2000 e^{0.025t} \cos t \dots (2)$$

Integral of  $\int 2000 e^{0.025t} \cos t$

$$\Rightarrow 2000 \int e^{0.025t} \cos t$$

$$\text{let } u = \cos t$$

$$du = -\sin t dt$$

$$du = -\sin t dt$$

$$v = 40 e^{0.025t} \downarrow$$

$$= 2000 (40 e^{0.025t} \cos t - \int 40 e^{0.025t} - \sin t dt)$$

$$= 80,000 e^{0.025t} \cos t + 80000 \int \sin t dt e^{0.025t}$$

$$= 80,000 e^{0.025t} \cos t + 1600 \int 50 \sin t dt e^{0.025t}$$

$$= 80,000 e^{0.025t} \cos t + 1600 \bar{I}$$

Using equation (2)

$$\bar{I} = 2000 \sin t e^{0.025t} - 80,000 e^{0.025t} \cos t - 1600 \bar{I}$$

$$1601 \bar{I} = 2000 \sin t e^{0.025t} - 80,000 e^{0.025t} \cos t$$

$$1601 \bar{I} = e^{\frac{t}{40}} (2000 \sin t - 80,000 \cos t)$$

$$1601 \bar{I} = \exp(t/40) \left( 80,000 \left( \frac{\sin t}{40} - \cos t \right) \right)$$



$$I = \exp(\tau/40) \left( \frac{\sin \tau - \cos \tau}{40} 80,000 \right)$$

$$\therefore 2000 \exp(\tau/40) + I = Q$$

$$\int Q \cdot IF = \exp(\tau/40) \left( 2000 + \frac{\sin(\tau) - \cos(\tau)}{40} 80,000 \right)$$

from  $y \cdot IF = \int Q \cdot IF$

$$y \cdot \exp(\tau/40) = \exp(\tau/40) \left( 2000 + \frac{\sin(\tau) - \cos(\tau)}{40} 80,000 \right)$$

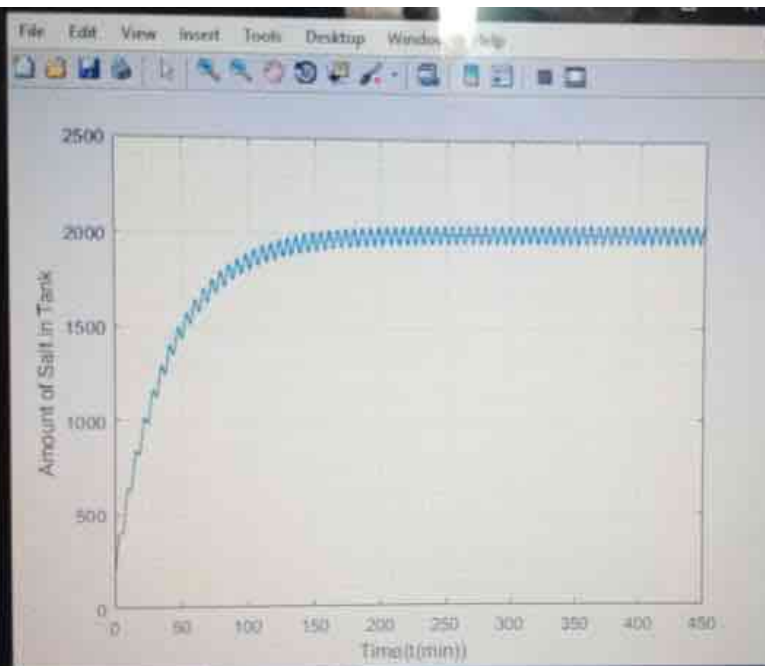
$$\therefore y = 2000 + \frac{\sin(\tau) - \cos(\tau)}{40} 80,000$$

```

1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - syms m t
6 - s=dsolve('Dm+(0.025*m) =50*(1+sin(t))','m(0)=150')
7 - tn=0:0.5:450
8 - sn=subs(s,tn)
9 - plot(tn,sn)
10 - grid on
11 - grid minor
12 - ylabel('Amount of Salt in Tank')
13 - xlabel('Time(t(min))')
14
15

```

1c)



```

1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - ta=2:2:500
6 - tb=1:2:500
7 - y=(50/0.05)+((50/1.0025)*sin(tb))+((50*0.05)/(1.0025))*cos(tb)-802.49*exp(-0.05*tb)
8 - ym=1000-(800*exp(-0.05*ta))
9 - yg=[y ym]
10 - tg=[ta tb]
11 - plot (tg,yg)
12 - grid on
13 - grid minor
14 - xlabel ('V(litre)')
15 - ylabel ('T(min)')
16 - heading1=('T(min)')
17 - heading2=('V(litre)')
18 - xlswrite('odevbesdata.xlsx',[tg(:),yg(:)],'variable','A1')
19 - xlswrite('odevbesdata.xlsx',heading1,'variable','A1')
20 - xlswrite('odevbesdata.xlsx',heading2,'variable','B1')

```

Command Window

473 475 477 479 481 483 485 487 489 491 493 495 497 499

```

1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - ta=2:2:500
6 - tb=1:2:500
7 - y=(50/0.05)+((50/1.0025)*sin(tb))+((50*0.05)/(1.0025))*cos(tb)-802.49*exp(-0.05*tb)
8 - ym=1000-(800*exp(-0.05*ta))
9 - yg=[y ym]
10 - tg=[ta tb]
11 - plot (tg,yg)
12 - grid on
13 - grid minor
14 - xlabel ('V(litre)')
15 - ylabel ('T(min)')
16 - heading1=('T(min)')
17 - heading2=('V(litre)')
18 - xlswrite('odevbesdata.xlsx',[tg(:),yg(:)],'variable','B2')
19 - xlswrite('odevbesdata.xlsx',heading1,'variable','A1')
20 - xlswrite('odevbesdata.xlsx',heading2,'variable','B1')

```

Command Window

473 475 477 479 481 483 485 487 489 491 493 495 497 499