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MATRIC NO: 18/ENG06/012
DEPARTMENT: MECHANICAL ENGINEERING
COURSE: ENGINEERING MATHEMATICS II
ASSIGNMENT

① 1200 gal of water
150lb of dissolved salt
50gal of brine, each gallon contains $(1 + \sin t)$ lb of dissolved salt
Rate = 30gal per minute
The given equation,

$$\frac{dm}{dt} = \frac{m}{40} + 50(1 + \sin(t))$$

It can be rewritten as;

$$\frac{dm}{dt} + \frac{m}{40} = 50(1 + \sin(t))$$

Using IF = IF = $e^{\int P\,dt}$

$$\int P\,dt = \int 1/40 = t/40$$

$$\therefore e^{\int P\,dt} = e^{t/40}$$

Recall that;

$$IF = \int Q\,IF$$

$$m \cdot e^{t/40} = \int (50(1 + \sin(t))) \cdot e^{t/40}$$

$$m \cdot e^{t/40} = 50\int (1 + \sin(t)) \cdot e^{t/40}$$

$$\therefore m \cdot e^{t/40} = 50\left(\int e^{t/40} + dt + \int e^{t/40} \sin(t)\,dt\right)$$

$$m \cdot e^{t/40} = 50(40e^{t/40} + \int e^{t/40} \sin(t)\,dt \quad \text{—— eqn ①}$$

$$\int u\,dv = uv - \int u\,dv$$

$u = \sin(t)$ $dv = e^{t/40}$

$du = \cos(t)$ $v = 40e^{t/40}$

$$\therefore \int e^{t/40} \sin(t)\,dt = 40e^{t} \times \sin t - \int 40e^{t/40} \cdot \cos(t)\,dt$$

I —— eqn ②

USING INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du$$

$u = \cos(t)$ $dv = 40e^{t/40}$, $du = -\sin(t)$ $v = 1600e^{t/40}$

$\int 40\, e^{t/40} \cdot \cos(t)\, dt = 1600\, e^{t/40} \cos(t) - \int -1600\, e^{t/40} \cdot \sin(t)$

$= 1600^{t/40} \cos(t) + 1600 \int e^{t/40} \sin(t)$ ——— eqn ③

We then substitute eqn ② into eqn ①

$\int e^{t/40} \sin(t)\, dt = 40\, e^{t/40} \sin(t) - (1600\, e^{t/40} \cos(t) + 1600 \int e^{t/40} \sin(t)\, dt$

$= 40\, e^{t/40} \cdot \sin(t) - 1600^{t/40} \cos(t) - 1600 \int e^{t/40} \sin t$

We the simplify the equation.

$\int e^{t/40} \sin(t)\, dt - 1600 \int e^{t/40} \sin(t)\, dt = 40\, e^{t/40} \sin t - 1600\, e^{t/40} \cos(t)$

$1601 \int e^{t/40} \sin(t)\, dt = 40\, e^{t/40} \sin t - 1600\, e^{t/40} \cos(t)$

$\int e^{t/40} \sin(t)\, dt = \dfrac{40\, e^{t/40} \sin(t) - 1600\, e^{t/40} \cos(t)}{1601}$

We then substitute eqn ③ into eqn ①

$m \cdot e^{t/40} = 50\left(\dfrac{40\, e^{t/40} + 40\, e^{t/40} \sin(t) - 1600\, e^{t/40} \cos(t)}{1601}\right)$

$m \cdot e^{t/40} = 2000\, e^{t/40} + \dfrac{2000\, e^{t/40} \sin(t) - 80000\, e^{t/40} \cos t + c}{1601}$

We then divide through by $e^{t/40}$

$m = 2000 + \dfrac{2000 \sin(t) - 80000 \cos(t)}{1601} + c\, e^{-t/40}$

$c = m_0$, When $t = 0$, $m = 150$lb

$150 = 2000 + \dfrac{2000 \sin(0) + 80000 \cos(0)}{1601}$

$150 = 2000 + -\left(\dfrac{80000}{1601} + m_0\right)$

$150 = 1950 + m_0$

$m_0 = 150 - 1950$

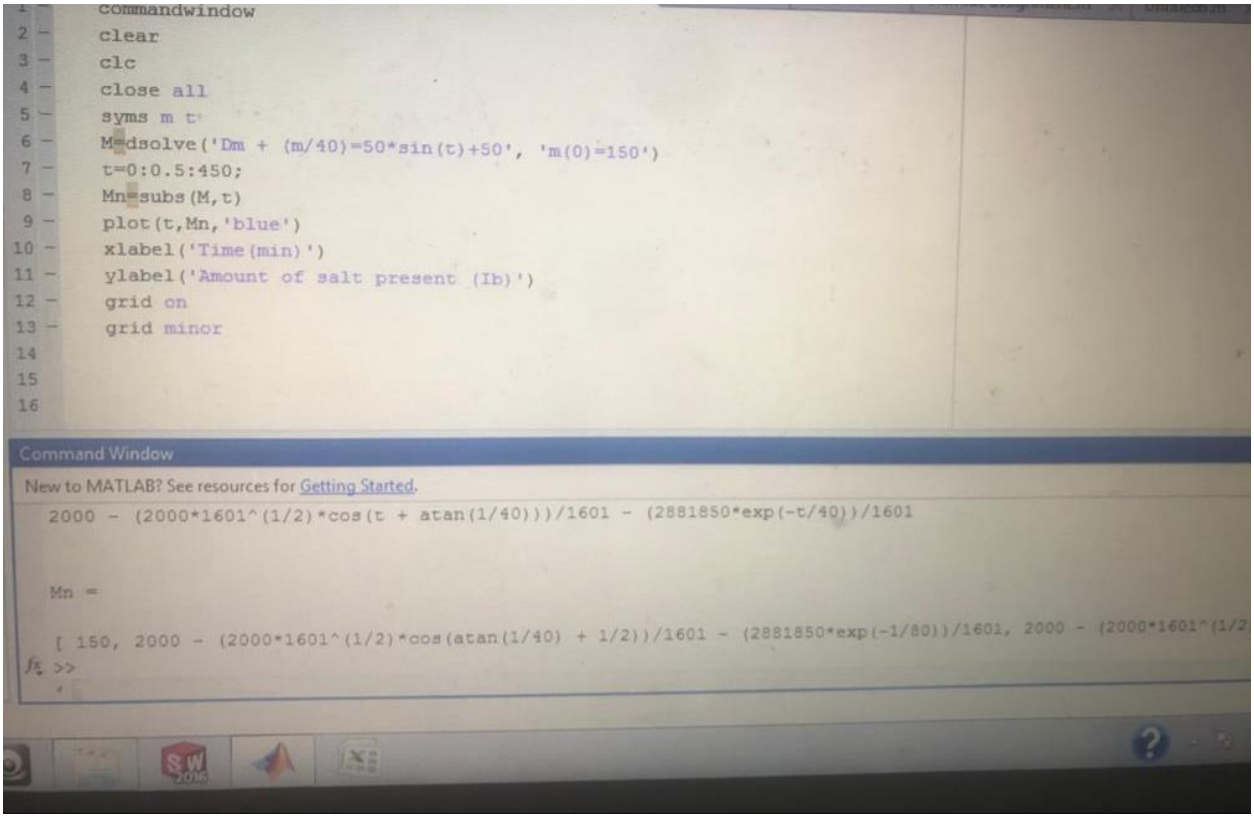
$m_0 = -1800$

Therefore, the equation for m is

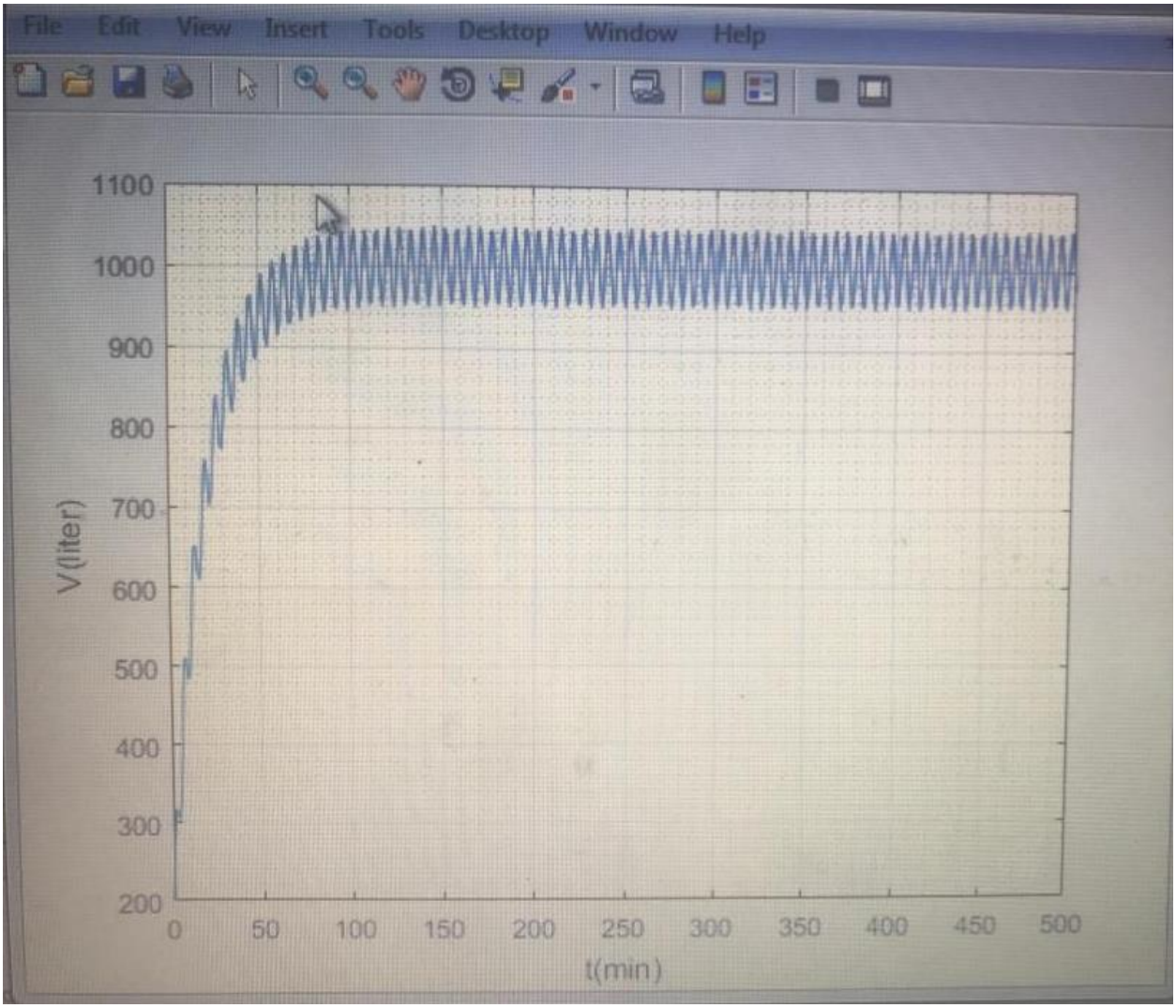
$$m = 2000 + \frac{2000\sin(t) - 80000\cos(t)}{1601} + 1800e^{-t/40}$$

The MATLAB mFile program to solve the differential equation

```
1  Commandwindow
2  clear
3  clc
4  close all
5  syms m t
6  m = dsolve('Dm + (m/40) = 50*sin(t) + 50', 'm(0) = 150')
7  t = 0 : 0.5 : 450;
8  mn = subs(M, t)
9  plot(t, min, 'blue')
10 xlabel('Time (min)')
11 ylabel('Amount of salt present (lb)')
12 grid on
13 grid minor
```


commandwindow
clear
clc
close all
syms m t
M=dsolve('Dm + (m/40)=50*sin(t)+50', 'm(0)=150')
t=0:0.5:450;
Mn=subs(M,t)
plot(t,Mn,'blue')
xlabel('Time(min)')
ylabel('Amount of salt present (lb)')
grid on
grid minor
Command Window
New to MATLAB? See resources for Getting Started.
2000 - (2000*1601^(1/2)*cos(t + atan(1/40)))/1601 - (2881850*exp(-t/40))/1601
Mn =
[150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, 2000 - (2000*1601^(1/2
fx >>

```
commandwindow
clear
clc
close all
syms t
y = (50/0.05)+((50/1.0025)*sin(t))+(((50*(0.05))/1.0025)*cos(t
ym = 1000-(800*exp(-0.05*t))
oddValues = 1:2:500
evenValues = 2:2:500
ym = double(subs(y, oddValues))
ymm = double (subs(ym, evenValues))
totTime = 1:1:500
timeTrans = totTime'
c = reshape([ym,ymm],[],1)
combVal = double(c)
plot(totTime, c)
grid on
grid minor
xlabel('T(min)'), ylabel('V(litre)')
col_header = {'t(min)', 'V(Litre)'}
xlswrite['odevbesdata.xlsx',col_header,'veriler','A2')
xlswrite['odevbesdata.xlsx',timeT,'veriler','A3')
xlswrite['odevbesdata.xlsx',combined,'veriler','B2')
```


File Edit View Insert Tools Desktop Window Help
1100
1000
900
800
700
600
500
400
300
200
V(liter)
0 50 100 150 200 250 300 350 400 450 500
t(min)