AME: OKWUOKWU BRYAN   COURSE: MAT 102
MATRIC NO.: 19/ENG05/049   DEPARTMENT: Mechatronics

) $A = 3i + 7j - 2k \quad B = i + 3j + 7k \quad C = 9i - 4j + 6k$

(i) $\cos\theta = \dfrac{A \cdot C}{|A||C|}$ $\quad A \cdot C = (3\times9) + (7\times-4) + (-2\times6)$

$= 27 + (-28) + (-12)$

$= -13$

$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{9 + 49 + 4} = \sqrt{62}$

$|C| = \sqrt{(9)^2 + (-4)^2 + (6)^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$

$\therefore \cos\theta = \dfrac{-13}{\sqrt{62}\sqrt{133}} \quad \theta = \cos^{-1}\left(\dfrac{-13}{\sqrt{62} \times \sqrt{133}}\right) = 98.23^\circ$

$\therefore$ Angle between $\vec{A}$ and $\vec{C}$ = 98.23°

(ii) $\cos\theta = \dfrac{B \cdot C}{|B||C|}$ $\quad B \cdot C = (1\times9) + (-3\times4) + (7\times6)$

$= 9 + (-12) + 42 = 39$

$|B| = \sqrt{(1)^2 + (3)^2 + (7^2)} = \sqrt{49 + 9 + 1} = \sqrt{59}$

$|C| = \sqrt{133}$

$\therefore \cos\theta = \dfrac{39}{\sqrt{59} \times \sqrt{133}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{39}{\sqrt{59} \times \sqrt{133}}\right)$

$\therefore$ Angle between B and C = 63.879°

(iii) $A + B + C = 3i + 7j - 2k + i + 3j + 7k + 9i - 4j + 6k$

$= 3i + i + 9i + 7j + 3j - 4j - 2k + 7k + 6k$

$\therefore A+B+C = 13i + 6j + 11k$

$u = \dfrac{A+B+C}{|A+B+C|}$ $\quad |A+B+C| = \sqrt{13^2+6^2+11^2}$

$= \sqrt{326}$

$u = \dfrac{13i+6j+11k}{\sqrt{326}} = \left(\dfrac{13}{\sqrt{326}}i, \dfrac{6j}{\sqrt{326}}, \dfrac{11k}{\sqrt{326}}\right)$

(2) $s = xi + yj + zk = (8t^2)i + (t^2-4t)j + (t+1)k$

velocity $= \dfrac{ds}{dt} = 16t\,i + (2t-4)j + k$

acceleration $= \dfrac{d^2s}{dt^2} = 16 + 2 + 0 = \underline{\underline{18 m/s^2}}$

(3) $A = 4i + 2j - 4k \quad B = 8i - 2j + k \quad C = i + 4j - 3k$

$(A \times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix} = i(2-8) - j(4+32) + k(-8+16)$
$= -6i - 36j + 8k$

$(A\times B)\times C = \begin{vmatrix} i & j & k \\ -6 & -36 & 8 \\ 1 & 4 & -3 \end{vmatrix} = i(108-32) - j(18-8)$
$+ k(-24+36)$

$\therefore (A \times B) \times C = 76i - 10j + 12k$