

MAT 102 assignments

1.) IF $A = 4i + j - 2k$, $B = 3i - 2j + k$ and $C = i - 2k$, Find

(a) $(A - 2B) \times C$

Where $2B = 2(3i - 2j + k) = 6i - 4j + 2k$

$(A - 2B) \times C$

$A - 2B = 4i + j - 2k - (6i - 4j + 2k)$

$= -2i + 5j - 4k \quad \times C$

$= \begin{matrix} i & j & k \\ -2 & 5 & -4 \end{matrix}$

$= i \begin{vmatrix} 5 & -4 \\ 0 & 2 \end{vmatrix} - j \begin{vmatrix} -2 & -4 \\ 1 & -2 \end{vmatrix} + k \begin{vmatrix} -2 & 5 \\ 1 & 0 \end{vmatrix}$

$= (10 - 0)i - (4 + 4)j + (0 - 5)k$

$= 10\hat{i} - 8\hat{j} - 5\hat{k}$

b) $A \times (2C \times 3B)$

Where $2C = 2(i - 2k) = 2i - 4k$

$3B = 3(3i - 2j + k) = 9i - 6j + 3k$

$2C \times 3B = \begin{matrix} i & j & k \\ 2 & 0 & -4 \\ 9 & -6 & 3 \end{matrix}$

$= i \begin{vmatrix} 0 & -4 \\ -6 & 3 \end{vmatrix} - j \begin{vmatrix} 2 & -4 \\ 9 & 3 \end{vmatrix} + k \begin{vmatrix} 2 & 0 \\ 9 & -6 \end{vmatrix}$

$= (0 - 24)i - (6 + 36)j + (-12 - 0)k$

$= -24i - 42j - 12k$

$A \times (-24\hat{i} - 42\hat{j} - 12\hat{k}) = \begin{matrix} i & j & k \\ 4 & 1 & -2 \\ -24 & -42 & -12 \end{matrix}$

$= i \begin{vmatrix} 1 & -2 \\ -42 & -12 \end{vmatrix} - j \begin{vmatrix} 4 & -2 \\ -24 & -12 \end{vmatrix} + k \begin{vmatrix} 4 & 1 \\ -24 & -42 \end{vmatrix}$

$= (84 + 12)i - (48 + 48)j + (-168 + 24)k$

$= 72\hat{i} - 96\hat{j} - 144\hat{k}$

2) $A = p\hat{i} - 6\hat{j} - 3\hat{k}$, $B = 4\hat{i} + 3\hat{j} - \hat{k}$ and $C = \hat{i} - 3\hat{j} + 2\hat{k}$. Find the value of p for which A , B and C are coplanar.

Solution

Three vectors \vec{a} , \vec{b} , \vec{c} are coplanar if

$$[\vec{a} \ \vec{b} \ \vec{c}] = 0$$

Given, $\vec{a} = p\hat{i} - 6\hat{j} - 3\hat{k}$

$$\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{c} = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= p(6-3) + 6(8+1) - 3(-12-3)$$

$$= 3p + 54 - 45 = 0$$

$$3p + 9 = 0$$

$$3p = -9$$

$$p = \frac{-9}{3} = -3$$

\therefore The value of $p = -3$.