

$$\frac{2000e^{-t/40}}{1601} \times (\sin t - 40 \cos t + 1601) + C$$

Substituting back into eqn (4)

$$m \dot{x} = \frac{2000e^{-t/40}}{1601} (\sin t - 40 \cos t + 1601) + C$$

$$m(t) = \frac{2000}{1601} \times (\sin t - 40 \cos t + 1601) + C e^{-t/40}$$

m is a function of t , and because there is 150 lb of salt with a time rate of 8 min

$$m(0) = \frac{2000}{1601} \times (\sin(0) - 40 \cos(0) + 1601) + C e^{-0/40}$$

$$150 = \frac{2000 \times 1561}{1601} + C$$

$$C = 150 - 1950.03$$

$$C = -1800.03$$

AREVA SOLUTIONS - EMPOWER

18/11/2015

(JEE ENGINEERING)

$$\frac{du}{dt} = \frac{1}{40}$$

$$dt = 40 du$$

Substitute for dt in (10)

$$2000 \int e^u (1 + \sin 40u) du$$

Integrating by parts,

$$\int u dv = uv - \int v du$$

$$\text{Choosing } u = \sin 40u, \quad dv = e^u$$

$$du = 40 \cos 40u, \quad v = e^u$$

$$= 2000 [e^u (\sin 40u + 1) - \int 40e^u \cos 40u \cdot du] \quad (11)$$

Solving for $\int e^u \cos 40u \cdot du$

Integrating by part,

$$\int u dv = uv - \int v du$$

$$= e^u \cos 40u - (-40e^u \sin 40u - \int -1600e^u \cos 40u)$$

$$= e^u \cos 40u - (-40e^u \sin 40u + 1600 \int e^u \cos 40u)$$

Solving for $\int e^u \cos 40u \cdot du$ the second time

$$= \frac{40 e^u \sin 40u + e^u \cos 40u}{1601}$$

1601

reinsert, $\int e^u \cos 40u \cdot du$ --- eqn (11)

$$\text{Therefore, } = 40 \left[\frac{40 e^u \sin 40u + e^u \cos 40u}{1601} \right]$$

Substituting from eqn (11)

$$= 2000 [e^u (\sin 40u + 1) - 40 \left(\frac{40 e^u \sin 40u + e^u \cos 40u}{1601} \right)]$$

$$= 2000 e^{t/40} (\sin t + 1) - 8000 \left(\frac{40 e^{t/40} \sin t + e^{t/40} \cos t}{1601} \right)$$

$$= 2000 e^{t/40} (\sin t + 1) - \frac{40 (40 \sin t \cos t) + C}{1601}$$

$$= 2000 e^{t/40} (\sin t + 1) - \frac{1600 \sin^2 t - 40 \cos t}{1601} + C$$

$$= \frac{2000 e^{t/40}}{1601} \times \frac{(1601 \sin t - 1600 \sin^2 t - 40 \cos t + 1601)}{1601} + C$$

APM 101: Mass-Action Equations
 18.11.2015
 Control Systems Response

From balance law:

Total rate of salt into the tank = Input rate of salt - output rate of salt + salt out of the tank

$$\frac{dQ}{dt} = \text{Input rate} - \text{output rate} \dots (1)$$

$$\text{Input rate} = 50 \text{ gal/min} \times (1 + \sin t) \text{ lb/gal}$$

$$= 50(1 + \sin t) \text{ lb/min} \dots (2)$$

$$\text{Output rate} = \frac{30 \text{ gal}}{125 \text{ gal}} \times 60 \times 2.5 \text{ lb/gal} \dots (3)$$

Assuming equation (1) = 0

$$\frac{dQ}{dt} = 50(1 + \sin t) - \frac{30}{125} \times 60 \times 2.5 \times 1$$

$$\frac{dQ}{dt} = 50(1 + \sin t) - 0.025m$$

$$*) \cdot \frac{dQ}{dt} - m/40 = 50(1 + \sin t) \dots (4)$$

5) Solve using integrating factor (IF)

$$\frac{d}{dt}(Q + I_1) = 0 \dots (5)$$

Integrating equation (4) with (5)

$$P = \frac{1}{40}, \quad Q = 50(1 + \sin t)$$

$$\text{IF} = e^{\int P dt} = \int \frac{1}{40} dt = \frac{t}{40}$$

$$\text{IF} = e^{t/40}$$

$$Q \cdot \text{IF} = \int Q \cdot \text{IF} \cdot dt$$

$$Q e^{t/40} = \int 50 e^{t/40} (1 + \sin t) dt \dots (6)$$

$$\text{Substituting } \frac{d}{dt} \left[\frac{500}{125} e^{t/40} (1 + \sin t) \right] dt \dots (7)$$

$$\text{Let } \frac{1}{125} = u, \quad \frac{d}{dt} = \frac{1}{40} \frac{d}{du}$$

Arithmetic progression

18 terms
 10th term = 100

From above law

Total sum of AP = $\frac{n}{2} [2a + (n-1)d]$
 = $\frac{18}{2} [2a + 17d]$
 = $9 [2a + 17d]$

10th term = $a + 9d = 100$... (i)

Total sum = $\frac{n}{2} [2a + (n-1)d]$
 $9 [2a + 17d] = 1000$... (ii)

From (i) $a = 100 - 9d$... (iii)

Substituting in (ii)

$9 [2(100 - 9d) + 17d] = 1000$
 $9 [200 - 18d + 17d] = 1000$
 $9 [200 - d] = 1000$
 $1800 - 9d = 1000$
 $800 = 9d$
 $d = \frac{800}{9}$

From (iii) $a = 100 - 9 \times \frac{800}{9}$
 $a = 100 - 800$
 $a = -700$

1) $\frac{dS}{dt} = m/40 = 50(1+50t)$... (iv)

2) Solve using integrating factor (IF)

$\frac{dS}{dt} + P_1 S = Q_1$... (v)

Integrating equation (iv) with (v)

$P = \frac{1}{40}$, $Q = 50(1+50t)$

IF = $e^{\int P dt}$
 $= e^{\int \frac{1}{40} dt}$
 $= e^{t/40}$

$S e^{t/40} = \int Q \cdot IF \cdot dt$

$S e^{t/40} = \int 50 e^{t/40} (1+50t) dt$... (vi)

Solving the (vi)
 $50 \int e^{t/40} (1+50t) dt$... (vii)
 Let $t/40 = u$; $t = 40u$

APPS EDITOR PUBLISH VIEW

Insert Comment Indent Breakpoints Run Run and Advance Run Section Advance Run as Time

NAVIGATE EDIT BREAKPOINTS RUN

Users > Tobiloba > Documents >

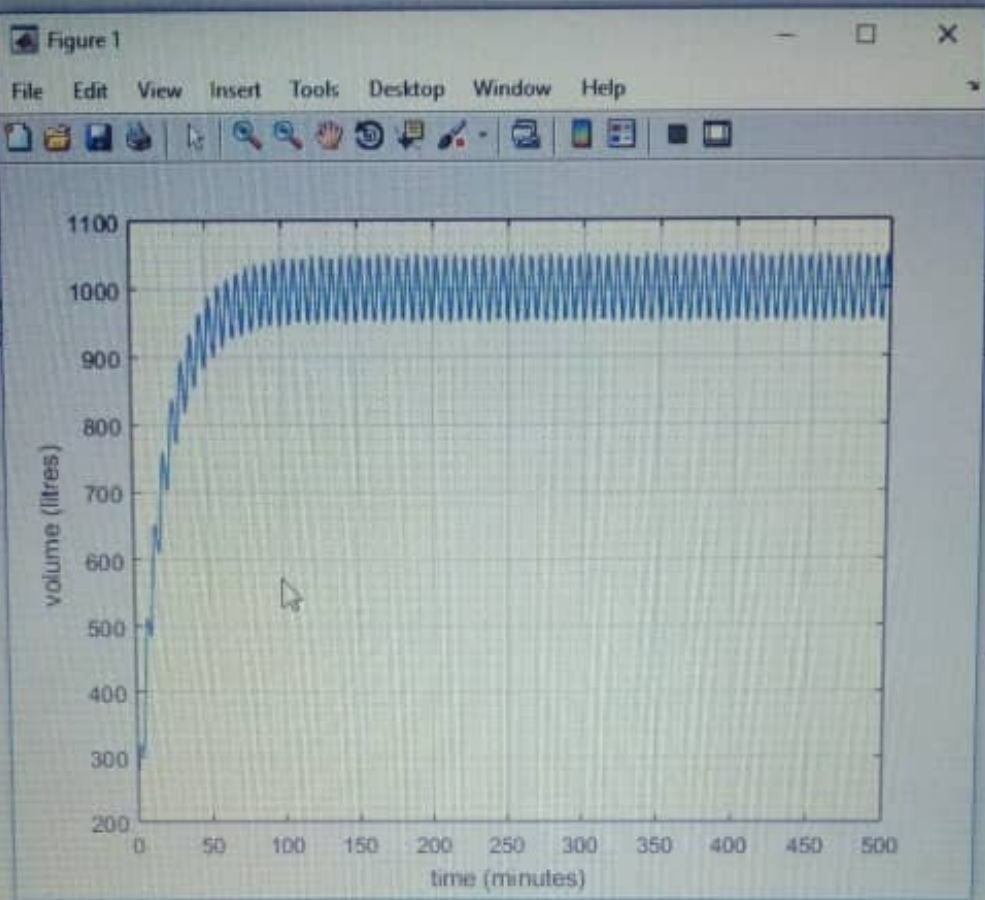
Editor - C:\Users\Tobiloba\Documents\TOBI.m

Command Window

New to MATLAB? See resources for [Getting Started](#).

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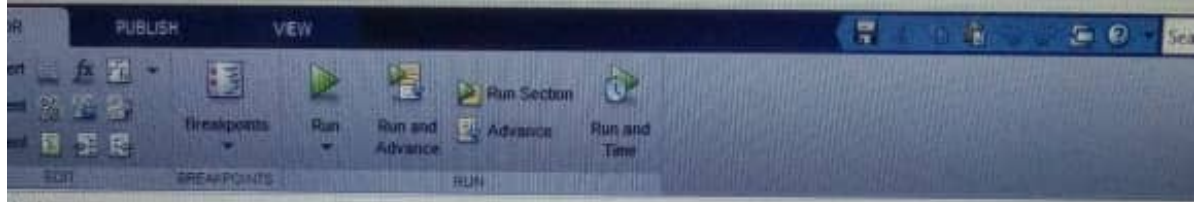
Taskbar icons: File Explorer, Microsoft Word, MATLAB, Mail, Browser, etc.



```
16 - plot(s, x)
19 - grid minor
20 - grid on
21
22 - xlabel('time (minutes)')
23 - ylabel('volume (litres)')
24 - xlsxwrite('odevbeedata.xlsx', {'time (minutes)', 'volume (litres)'}, 'A1')
25 - xlsxwrite('odevbeedata.xlsx', mins, 'volume (litres)', 'A2')
```

Command Window





Editor - C:\Users\Fobitoba\Documents\Y081.m

Command Window

New to MATLAB? See resources for [Getting Started](#)

```
Columns 81 through 100
    81    82    83    84    85    86    87    88    89    90    91    92    93    94    95    96

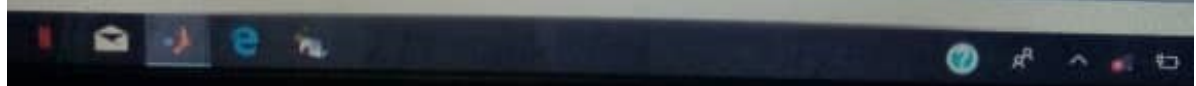
Columns 101 through 120
   101   102   103   104   105   106   107   108   109   110   111   112   113   114   115   116

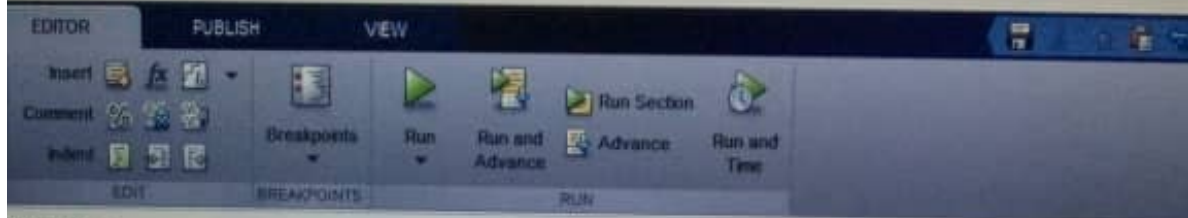
Columns 121 through 140
   121   122   123   124   125   126   127   128   129   130   131   132   133   134   135   136

Columns 141 through 160
   141   142   143   144   145   146   147   148   149   150   151   152   153   154   155   156

Columns 161 through 180
   161   162   163   164   165   166   167   168   169   170   171   172   173   174   175   176

Columns 181 through 200
   181   182   183   184   185   186   187   188   189   190   191   192   193   194   195   196
```





Documents

```
Editor - C:\Users\Tobiloba\Documents\TOBI.m
TOBI.m x +
1 - commandwindow
2 - clear
3 - close all
4
5 - syms a
6
7 - x = []
8 - a = 1:1:500
9 - y1 = 1000 - ((exp(-0.05*a))*800)
10 - y = 1000 + (50/1.0025)*sin(a) + (2.5/1.0025)*cos(a) - ((exp(-0.05*a))*802.49)
11 - if rem(a,2) == 0
12 -     x = [x, y1]
13 - else
14 -     x = [x, y]
15 - end
16 - TOBIAM = transpose(x)
17 - mins = transpose(a)
18 - plot(a, x)
19 - grid minor
20 - grid on
21
22 - xlabel('time (minutes)')
23 - ylabel('volume (litres)')
24 - xiswrite('derivdata.xlsx', [1:500], 'volume', 'A1')
25 - xiswrite('derivdata.xlsx', mins, 'time', 'A1')
Command Window
```

