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18/ENG01/004
Engineering maths
Chemical Engineering

→ A tank contains 1200 gal of water
Mass of salt = 150 lb of salt
Volume input = 50 gal of brine
(1 + sin t) lb of salt/min = mass output of salt
Volume output = 30 gal/min
From balance law.
accumulating rate = Input rate of salt - Output rate of salt
Let mass input rate be = m_1
Let mass output rate be = m_2

$\therefore \frac{dm}{dt} = m_1 - m_2$ — (i)

$\therefore m_1 = \frac{50\,gal}{min} \times \frac{(1+\sin t)\,lb}{gal}$

$= 50(1+\sin t)$ lb/min — (ii)

$\therefore m_2 = \frac{30\,gal}{1200\,gal} \times 100 = 2.5\%$ of m — (iii)

Then we put eq(ii) and (iii) into eq (i)

$\frac{dm}{dt} = 50(1+\sin t) - \frac{2.5}{100} \times m$

$= 50(1+\sin t) - 0.025m$

$\frac{dm}{dt} = 50(1+\sin t) - \left(\frac{1}{40}\right)m$

a → $\frac{dm}{dt} + \left(\frac{1}{40}\right)m = 50(1+\sin t)$ — (iv)

b → Using the Integrating factor (I_t)

$\frac{dy}{dx} + Py = Q$ — (v)

Relate coeff. in equations (iv) and (v)

$P = \frac{1}{40}$ $\quad Q = 50(1+\sin t)$ $\quad I_F = e^{\int P dt}$

$\int P dt = \int 1/40 dt \therefore \frac{t}{40}$ $\therefore I_F = e^{t/40}$

$n \cdot I_f = \int Q \cdot I_f dt$

$ne^{t/40} = \int 50e^{t/40}(1+\sin t)dt$ —— (VII)

For the RHS,

$50\int e^{t/40}(1+\sin t)dt$ —— (VII)

Let $t/40 = u$ $\therefore t = 40u$

then $\frac{du}{dt} \times \frac{1}{40}$ $dt = 40du$

Put $dt = 40du$ into equation (VII)

$50\int e^{t/40}(1+\sin t) \times 40 du = 2000 \int e^u (1+\sin 40u) du$

Then using Integration by part,

$\int u dv = uv - \int v du$

let $u = \sin 40u$ $\frac{1}{2}du = 40\cos 40u$

$v = e^u$ $dv = e^u$

$\therefore 2000[e^u(\sin 40u + 1) - \int 40e^u \cos 40u - du$ —— (VIII)

Using integration by part;

$\int e^u \cos 40u \cdot du$

$\int u dv = uv - \int v du$

$u = \cos 40du$, $\therefore dv = e^u$ $v = e^u$

$u = -40\sin 40u$ $du = -1600\cos 40u$

$\therefore e^u \cos 40u - (-40e^u \sin 40u - \int -1600 e^u \cos 40u du$ — (x)

$\therefore e^u \cos 40u - (-40e^u \sin 40u + 1600 \int e^u \cos 40 du$ —— (x)

for $\int e^u \sin 40u du$

$\frac{40e^u \sin 40u + e^u \cos 40u}{1601}$ —— (xi)

Remember, $\therefore \int e^u \cos 40u \cdot du$

$\therefore 40\left(\frac{-40e^u \sin 40u + e^u \cos 40u}{1601}\right)$

When put in equation (VIII)

$$2000\left[\left[e^{u}(\sin 40u+1)-40\left(\frac{40e^{u}\sin 40u+e^{u}\cos 40u}{1601}\right)\right]+C\right.$$

$$2000e^{u}(\sin 40u+1)-80000\,\frac{(40e^{u}\sin 40u+e^{u}\cos 40u}{1601}+C$$

$u = t/40$

$$2000e^{t/40}(\sin t+1)-80000\,\frac{(40e^{t/40}\sin t+e^{t/40}\cos t)}{1601}+C$$

$$2000e^{t/40}\left[(\sin t+1)-80000e^{t/40}-(40\sin t)-\cos t\right.+C$$

$$2000e^{t/40}\left[(\sin t+1-40(40\sin t+\cos t)+\right.$$

$$2000e^{t/40}\left[(\sin t+1)-1600\sin t-40\cos t\right)+C$$

$$2000^{t/40}\,(1601\sin t-1600\sin t-40\cos t+1601)+C$$

$$2000e^{t/40}\times(\sin t-40\cos t+1600)+C$$

Put into equation (VI)

$$m e^{t/40}=\frac{2000e^{t/40}}{1601}(\sin t-40\cos t+1601)+c$$

$$m(t)=\frac{2000}{1601}+(\sin t-40\cos t+1601)+C$$

∴ m remains a function of t

150lb of salt in the tank at 0mins $= t$

$$m(0)=\frac{2000}{1601}\times\sin(0)-40\cos(0)+1601)+C$$

$$\therefore 150=\frac{2000}{1601}(-40+1601)+C$$

$$\therefore 150=\frac{2000\times 1561}{1601}+C \quad \therefore C=150-1950.029$$

$$C=-1800.029$$

∴ The expression of the tank would be;

$$m(t)=\frac{2000}{1601}(\sin t-40\cos t+1601)-\frac{1800}{e^{t/40}}\text{ lb}$$

```
1 -  commandwindow
2 -  clear
3 -  clc
4 -  close all
5 -  t=1:2:500;
6 -  y=((50/0.05) + ((50/1.0025)*sin(t)) + (((50*0.05)/1.0025)*cos(t)) - (802.49*exp(-0.05*t)))
7 -  tm=0:2:500;
8 -  ym=((1000)-(800*exp(-0.05*tm)))
9 -  T=[t,tm]
10 - Y=[y,ym]
11 - plot(T,Y,'green')
12 - grid on
13 - grid minor
14 - xlabel('T(min)')
15 - ylabel('V(litre)')
16 - col_header={'t(min)','V(Litre'};
17 - xlswrite('odevbesdata.xlsx',[T(:),Y(:)],'veriler','A2');
18 - xlswrite('odevbesdata.xlsx',col_header,'veriler','A1');
```

Command Window

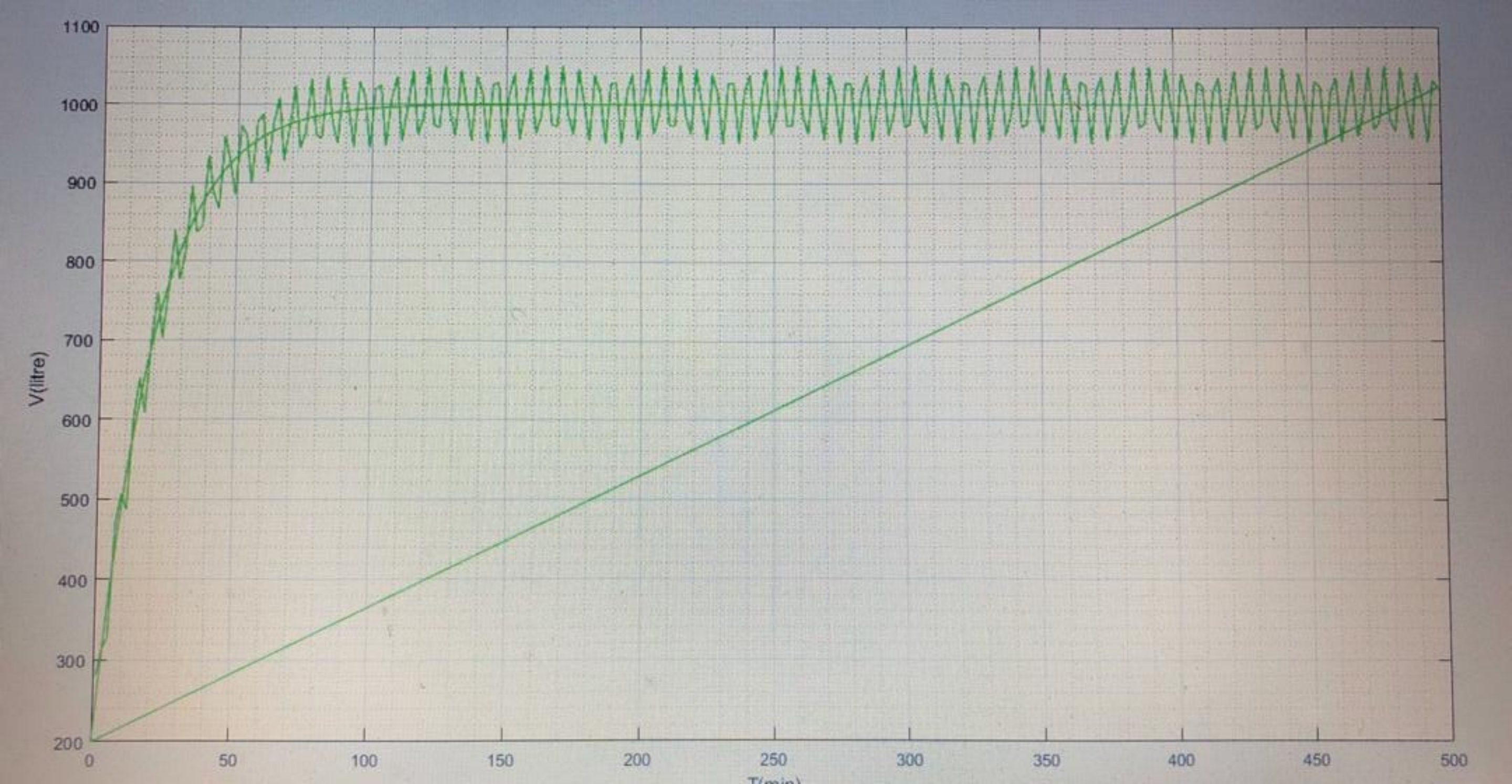
```
  Columns 481 through 492

    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000

  Columns 493 through 501

    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000

fx >>
```


1100
1000
900
800
700
600
500
400
300
200
V(litre)
0
50
100
150
200
250
300
350
400
450
500

```
1 -  commandwindow
2 -  clear
3 -  clc
4 -  close all
5 -  syms m t
6 -  ans=dsolve('Dm+0.025*m=50+50*sin(t)','m(0)=150')
7 -  t=0:0.5:450
8 -  tn=subs(ans,t)
9 -  plot(t,tn)
```

Command Window

New to MATLAB? See resources for Getting Started.

```
   445.5000   446.0000   446.5000   447.0000   447.5000   448.0000   448.5000   449.0000   449.5000   450.0

tn =

[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, 2000 - (20
fx >>
```

script