NAME: OKWUOKWU BRYAN COURSE: MAT 104

MATRIC NO.: 19/ENG05/1049 DEPARTMENT: Mechatronics

(1) (a) $y = \frac{3}{x^2}$ $\quad y + \Delta y = \frac{3}{(x+\Delta x)^2}$

$\Delta y = \frac{3}{x^2 + 2x\Delta x + \Delta x^2} + \frac{3}{x^2}$

$\Delta y = 3(x^2 + 2x\Delta x + \Delta x^2)^{-1} + 3(x^2)^{-1}$

(b) $y + \Delta y = \frac{4}{(x+\Delta x)^3} = \frac{4}{x^3 + 3\Delta x x^2 + 3x\Delta x^2 + \Delta x^3}$

$\Delta y = \frac{4}{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3} - \frac{4}{x^3}$

$\Delta y = \frac{4x^3 - 4(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}{x^3(x^3 + 3x^2\Delta x + 3\Delta x^2 x + \Delta x^3}$ =

$\frac{4x^3 - 4x^3 - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^6 + 3x^5\Delta x + 3\Delta x^2 x^4 + x^3\Delta x^3}$

$\frac{\Delta y}{\Delta x} = \frac{-12x^2\Delta x}{\Delta x} - \frac{12x\Delta x^2}{\Delta x} - \frac{4\Delta x^3}{\Delta x}$

$\frac{}{\Delta x} \quad x^6 + 3x^5\Delta x + 3\Delta x^2 x^4 + x^3\Delta x^3 \qquad (\Delta x \to 0)$

$\frac{dy}{dx} = \frac{-12x^2}{x^6} = \frac{-12}{x^4}$

(2) (a) $\int \frac{dx}{x^2+36} = \int \frac{dx}{x^2+6^2}$

$x = 6\tan\theta \qquad \frac{dx}{d\theta} = 6\sec^2\theta$

$dx = 6\sec^2\theta\, d\theta$

$x^2+6^2 = 6^2\tan^2\theta + 6^2 = 6^2(\tan^2\theta+1) = 36\sec^2\theta$

$\Rightarrow \int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{1}{6}\int d\theta$

$= \frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C$

(b) $\int \frac{dx}{x^2+13} = \int \frac{dx}{x^2+(\sqrt{13})^2}$

$x = \sqrt{13}\tan\theta \qquad \frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$

$dx = \sqrt{13}\sec^2\theta\, d\theta$

$x^2+(\sqrt{13})^2 = (\sqrt{13})^2\tan^2\theta + (\sqrt{13})^2 = \sqrt{13}^2(\tan^2\theta+1)$

$= 13\sec^2\theta$

$\Rightarrow \int \frac{\sqrt{13}\sec^2\theta\, d\theta}{13\sec^2\theta\, d\theta} = \int \frac{d\theta}{\sqrt{13}} = \frac{1}{\sqrt{13}}\int d\theta$

$= \frac{1}{\sqrt{13}}\tan^{-1}\left(\frac{x}{\sqrt{13}}\right) + C$