

Name: Katelyn Godwin

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ENC 282

A

1. Accumulator rate = input rate - output rate

$$\frac{dm}{dt} = m_{in} - m_{out}$$

$$m_{in} = (1 + 5mt) \frac{lb}{gallon} \times 50 \frac{gallons}{min} = 50(1 + 5mt) \frac{lb}{min}$$

$$m_{out} = \frac{30 \text{ gallons}}{1200 \text{ gallons}} \leftarrow 0.025 \text{ water runs out hence } 0.2$$

2. 0.025 gallons of water runs out hence 0.025 rate runs out
= 0.025m

$$\frac{dm}{dt} = 50(1 + 5mt) - 0.025m$$

$$\frac{dm}{dt} + 0.025m = 50(1 + 5mt)$$

Comparing to $\frac{dy}{dx} + P_y = Q$

$$dy = dm, dx = dt, P_y = 0.025, Q = 50(1 + 5mt)$$

∴ I.F =

$$m \cdot I.F = \int Q \cdot I.F dt \text{ where } I.F = e^{\int P dx}$$

$$\int P dx = \int 0.025 dt = 0.025t$$

$$I.F = e^{0.025t}$$

Integrating

$$m e^{0.025t} = \int 50(1 + 5mt) \cdot e^{0.025t} dt$$

$$m \cdot e^{0.025t} = 50 \int (e^{0.025t} + e^{0.025t} 5mt) dt$$

$$m e^{0.025t} = 50 \int \left(\frac{e^{0.025t}}{0.025} \right) + \int e^{0.025t} 5mt dt \quad \text{--- (1)}$$

$$\int e^{0.025t} 5mt dt = uv - \int v du$$

$$u = e^{0.025t} \quad dv = 5mt$$

$$v = 0.025e^{0.025t}$$

$$dv = 0.025e^{0.025t}$$

$$\int e^{0.025t} \sin t dt = e^{0.025t} (-\cos t) - \int -\cos t \cdot 0.025 e^{0.025t}$$

$$\int e^{0.025t} \sin t dt = -e^{0.025t} \cos t + 0.025 \int e^{0.025t} \cos t \dots (1)$$

Integrating again ($e^{0.025t} \cos t$)

$$\int e^{0.025t} \cos t = u \cdot v - \int v \cdot du$$

$$u = e^{0.025t} \quad dv = \cos t$$

$$du = 0.025 e^{0.025t} \quad v = \sin t$$

$$\int e^{0.025t} \cos t = e^{0.025t} \sin t - \int \sin t (0.025 e^{0.025t})$$

$$\int e^{0.025t} \cos t = e^{0.025t} \sin t - 0.025 \int e^{0.025t} \sin t \dots (2)$$

Substituting equation (2) into (1)

$$\int e^{0.025t} \sin t dt = -e^{0.025t} \cos t + 0.025 \left(e^{0.025t} \sin t - 0.025 \int e^{0.025t} \sin t dt \right)$$

$$\int e^{0.025t} \sin t dt = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t - 0.00625 \int e^{0.025t} \sin t dt$$

Grouping the like terms for both equations:

$$1.000625 \int e^{0.025t} \sin t dt = e^{0.025t} (-\cos t + 0.025 \sin t)$$

$$\int e^{0.025t} \sin t dt = \frac{e^{0.025t}}{1.000625} \{-0.025 \sin t - \cos t\} \dots (3)$$

Substituting eqn (3) into equation (1)

$$m e^{0.025t} = 50 \int \left\{ \frac{e^{0.025t}}{0.025} + \frac{e^{0.025t}}{1.000625} \{-0.025 \sin t - \cos t\} + c \right\}$$

$$m = 2000 + 49.969 \{-0.025 \sin t - \cos t\} + \frac{50c}{e^{0.025t}}$$

Recall that $m(0) = 150$

hence

$$150 = 2000 + 49.969 \{-0.025 \sin(0) - \cos(0)\} + \frac{50c}{e^{0.025(0)}}$$

$$150 = 2000 + 49.969(-1) + 50c$$

$$150 = 1950.031 + 50c$$

$$c = \frac{-1950.031 + 150}{50} = -36.0062$$

$$c = -36.0062$$

hence the final equation is

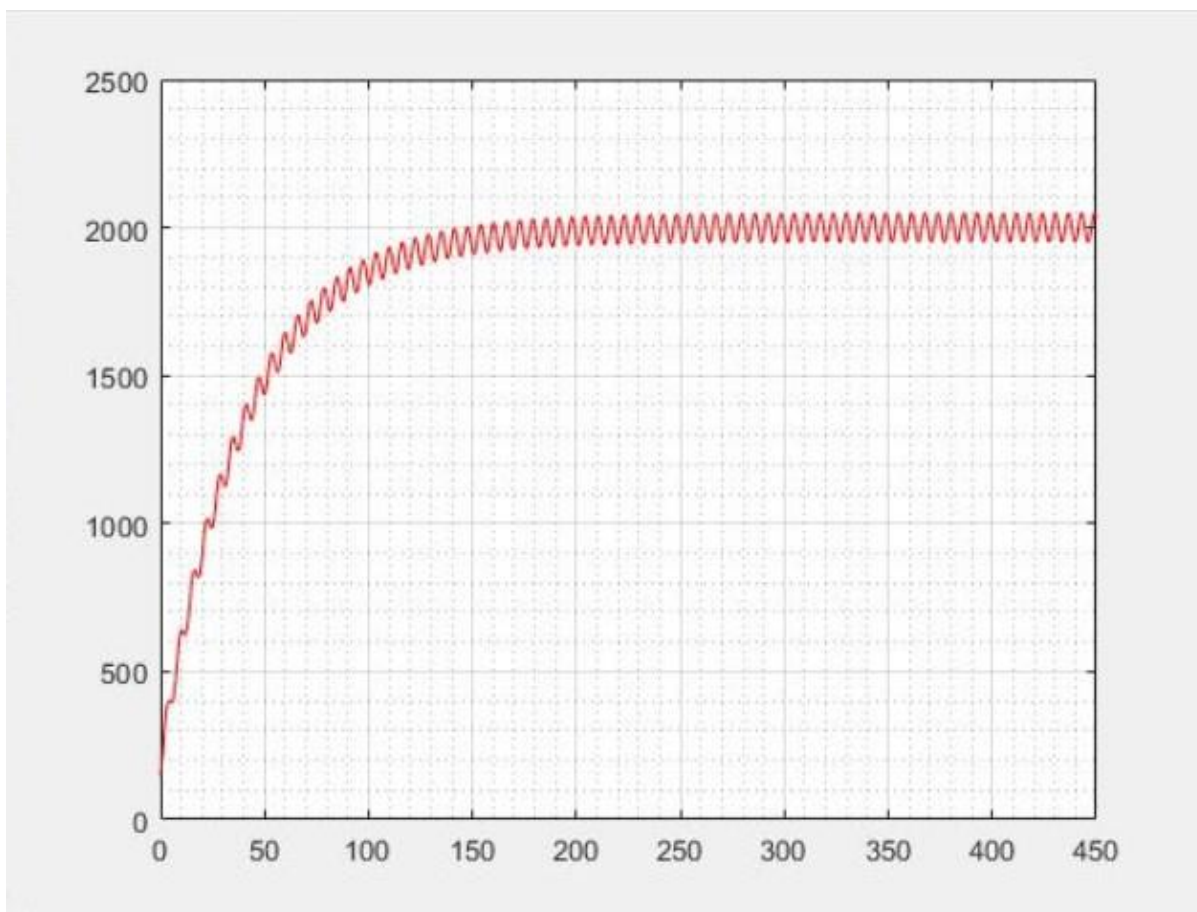
$$m = 2000 + \frac{50}{1.000625} \{-0.025 \sin t - \cos t\}$$

$$= 2000 - 36$$

$$m = 2000 + 1.2475t - 50 \ln t - 130e^{-0.025t}$$

```
1 - commandwindow
2 - clc
3 - close all
4 - syms y
5 - y=dsolve('Dy=(50*(1+sin(t))-(0.025*y))','y(0)=150')
6 - pretty(y)
7 - t=0:0.5:450
8 - yn=subs(y,t)
9 - plot(t,yn,'red')
10 - grid on
11 - grid minor
12
13
14
15
16
17
```

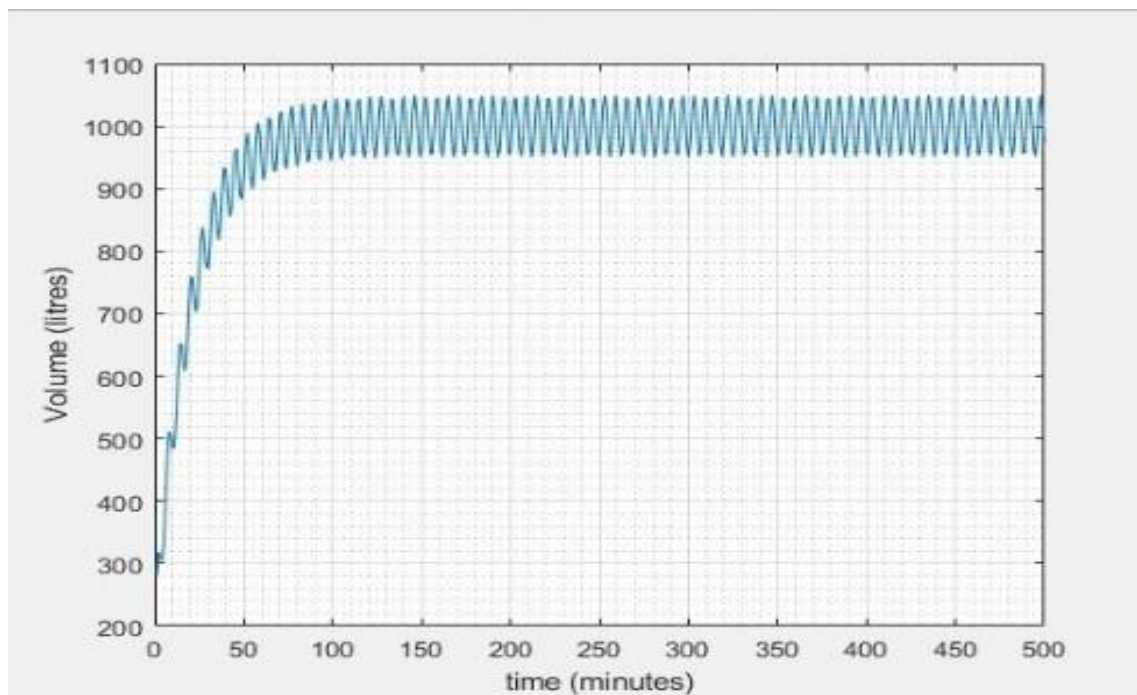
Graph



NUMBER 2

```
1 -  commandwindow
2 -  clear
3 -  clc
4 -  close all
5
6 -  syms time
7 -  x = [];
8 -  time = 1:1:500;
9 -  y = (50/0.05) + (50/1.0025)*sin(time) + (2.5/1.0025)*cos(time) - ((exp(-0.05*time
10 - ymean = 1000 - ((exp(-0.05*time))*800);
11 -  if rem(time,2) == 0
12 -      x = [x,ymean];
13 -  else
14 -      x = [x,y];
15 -  end
16 -  excelvalues = transpose(x);
17 -  mins = transpose(time);
18 -  plot(time, x)
19 -  grid on
20 -  grid minor
21 -  xlabel('time (minutes)')
22 -  ylabel('Volume (litres)')
23 -  xlswrite('odevbesdata.xlsx', {'time (min)'}, 'Veriler', 'A1')
24 -  xlswrite('odevbesdata.xlsx', {'V (litres)'}, 'Veriler', 'B1')
25 -  xlswrite('odevbesdata.xlsx', mins, 'Veriler', 'A2')
26 -  xlswrite('odevbesdata.xlsx', excelvalues, 'Veriler', 'B2')
```

GRAPH



Below are the values generated from Excel

First range of values:

t(min)	V(Litre)
1	279.9639
2	276.1301
3	313.8601
4	345.0154
5	327.9009
6	407.3454
7	469.1423
8	463.744
9	506.5922
10	514.7755
11	487.1398
12	560.9507
13	604.2824
14	602.7318
15	651.4694
16	640.5368
17	608.3676
18	674.7443
19	699.585
20	705.6964
21	759.541
22	733.7031
23	702.3679
24	759.0446

Last range of values:

477	977.2894
478	1000
479	1049.892
480	1000
481	981.1861
482	1000
483	965.767
484	1000
485	1047.306
486	1000
487	994.8607
488	1000
489	956.9717
490	1000
491	1040.952
492	1000
493	1008.945
494	1000
495	951.6039
496	1000
497	1031.335
498	1000
499	1022.316
500	1000