OBI-OBUOHA ABIAMAMELA

18/ENG05/040

MECHATRONICS ENGINEERING

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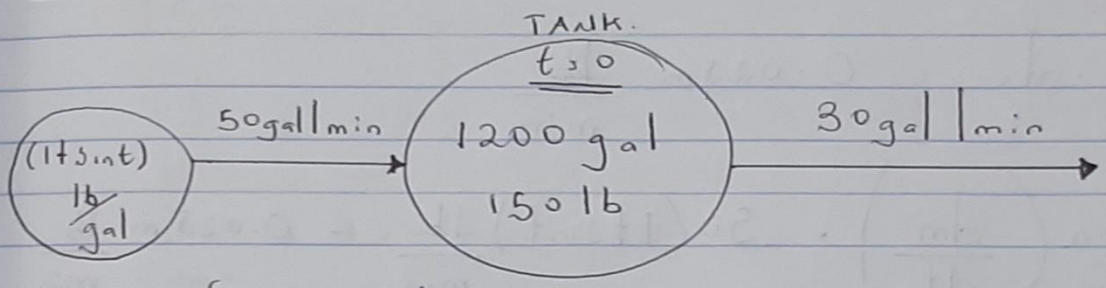
QUESTION 1.



Note: Accumulation rate = Input rate − Output rate

Input Rate

$(1+\sin t)\ \frac{lb}{gal}$ at 50 gal/min

$\therefore (1+\sin t)\ \frac{lb}{gal} \times 50\ \frac{gal}{min} = 50(1+\sin t)\ \frac{lb}{min}$

Amount of salt rate (Input) = $50(1+\sin t)\ \frac{lb}{min}$

Output Rate

30 gal/min ∴ for liquid

A/B: Since uniformity of mixture is ensured,

% output rate of water : % output rate of salt.

$$\frac{30\ \text{gal}}{1200\ \text{gal}}\Big/\text{min} = \frac{1}{40} \times 100\% = 2.5\%\ \text{per min}$$

$$2.5\% = \frac{2.5}{100} = 0.025\ \text{of m lb/min}$$

$\therefore$ Output rate $= 0.025m\ \frac{\text{lb}}{\text{min}}$

$$\text{Accumulation rate} \left(\frac{dm}{dt}\right) = 50(1+\sin t)\ \frac{\text{lb}}{\text{min}} - 0.025m\ \frac{\text{lb}}{\text{min}}$$

$$\frac{dm}{dt} = 50(1+\sin t) - 0.025m$$

$$\frac{dm}{dt} = -0.025m + 50(1+\sin t)$$

$$\frac{dm}{dt}$$

$$\frac{dm}{dt} = -0.025m + 50(1+\sin t)$$

$$\frac{dm}{dt} + 0.025m = 50(1+\sin t)$$

Comparing with $\frac{dy}{dx} + Py = Q$ $\quad P = 0.025$, $Q = 50(1+\sin t)$

∴ Use Integrating factor method.

$$IF = e^{\int P\,dx} = e^{\int 0.025\,dt} = e^{0.025t}$$

$$y \cdot IF = \int Q \cdot IF\,dx$$

$$= m \cdot IF = \int Q \cdot IF\,dt$$

$$m \cdot e^{0.025t} = \int Q \cdot e^{0.025t}\,dt$$

$$m \cdot e^{0.025t} = \int 50(1+\sin t)\,e^{0.025t}\,dt$$

$$m \cdot e^{0.025t} = 50 \int (1+\sin t)\,e^{0.025t}\,dt \quad -①$$

Integration of $50\int (1+\sin t)\,e^{0.025t}\,dt$

$u \quad\quad v \quad\quad dv$

$v = (1+\sin t)$

$dv = \cos t$

$du = e^{0.025t}$

$u = e^{0.025t}/0.025$

Integration by part.

$\int v\,du = uv - \int u\,dv$

$$\therefore \left(\frac{e^{0.025t}}{0.025}(1+\sin t)\right) - \frac{1}{0.025}\int e^{0.025t}\cos t$$

Integrate $e^{0.025t}\cos t$

Integration of $\int e^{0.025t}\cos t$

By parts

$\int e^{0.025t}\cos t$

$u = e^{0.025t}/0.025$; $du = e^{0.025t}$; $v = \cos t$; $dv = -\sin t$

$\therefore \int e^{0.025t}\cos t = \left(\frac{e^{0.025t}}{0.025}\cos t\right) - \frac{1}{0.025}\int e^{0.025t}(-\sin t)$

$\Rightarrow \left(\frac{e^{0.025t}}{0.025}\cos t\right) + \frac{1}{0.025}\int e^{0.025t}\sin t\, dt$

Integrate $e^{0.025t}\sin t$

$du = e^{0.025t}$; $u = e^{0.025t}/0.025$; $v = \sin t$; $dv = \cos t$

$\therefore \int e^{0.025t}\sin t = \left(\frac{e^{0.025t}}{0.025}\sin t\right) - \frac{1}{0.025}\int e^{0.025t}\cos t$

$\therefore \int e^{0.025t}\cos t$

$= \left(\frac{e^{0.025t}}{0.025}\cos t\right) + \frac{1}{0.025}\int \frac{e^{0.025t}}{0.025}\sin t - \frac{1}{0.025}\int e^{0.025t}\cos t$

$\int e^{0.025t}\cos t \Rightarrow \left(\frac{e^{0.025t}}{0.025}\cos t\right) + \frac{e^{0.025t}}{(0.025)^2}\sin t - \frac{1}{(0.025)^2}\int e^{0.025t}\cos t$

$\int e^{0.025t}\cos t + \frac{1}{(0.025)^2}\int e^{0.025t}\cos t = \left(\frac{e^{0.025t}}{0.025}\cos t\right) + \frac{e^{0.025t}}{(0.025)^2}\sin t$

$1601\int e^{0.025t}\cos t = \frac{e^{0.025t}}{0.025}\left(\cos t + \frac{\sin t}{0.025}\right)$

$$\int e^{0.025t} \cos t = \frac{1}{1601} \times \frac{e^{0.025t}}{0.025}\left(\cos t + \frac{\sin t}{0.025}\right)$$

$$\int e^{0.025t} \cos t = \frac{e^{0.025t}}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)$$

$$\therefore \int (1+\sin t)\, e^{0.025t}$$

$$\Rightarrow \left(\frac{e^{0.025t}}{0.025}(1+\sin t)\right) - \frac{1}{0.025}\left(\frac{e^{0.025t}}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)\right)$$

$$\therefore 50 \int (1+\sin t)\, e^{0.025t}$$

$$\Rightarrow \frac{50}{0.025}\left(\left(e^{0.025t}(1+\sin t)\right) - \left(\frac{e^{0.025t}}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)\right)\right)$$

$$2000\, e^{0.025}\left((1+\sin t) - \frac{1}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)\right)$$

A/B: $\cos t + \frac{\sin t}{0.025} = \frac{0.025\cos t + \sin t}{0.025}$

$$\therefore \Rightarrow 2000\, e^{0.025t}\left((1+\sin t) - \frac{1}{40.025}\left(\frac{0.025\cos t + \sin t}{0.025}\right)\right)$$

$$2000\, e^{0.025t}\left((1+\sin t) - \frac{1}{1.000625}\left(0.025\cos t + \sin t\right)\right)$$

∴ Put back in ①

$$m \cdot e^{0.025t} = 2000 e^{0.025t}\left((1+\sin t) - \frac{1}{1.000625}(0.025\cos t + \sin t)\right) + c$$

Dividing through by $e^{0.025t}$

$$m = 2000\left((1+\sin t) - \frac{1}{1.000625}(0.025\cos t + \sin t)\right) + \frac{c}{e^{0.025t}}$$

Initially when $t = 0$, $m = 150$ lb

$$\therefore 150 = 2000\left((1+\sin(0)) - \frac{1}{1.000625}(0.025\cos(0) + \sin(0))\right) + \frac{c}{e^{0.025(0)}}$$

$$150 = 2000\left((1+0) - \frac{1}{1.000625}(0.025(1) + 0)\right) + \frac{c}{e^0}$$

$$150 = 2000\left(1 - \frac{1}{1.000625}(0.025)\right) + \frac{c}{1}$$

$$150 = 2000\left(1 - \frac{0.025}{1.000625}\right) + c$$

$$c = 150 - 2000\left(1 - \frac{0.025}{1.000625}\right)$$

$$c = 150 - 1950.0312304$$

$$\therefore C \Rightarrow -1800.0312304$$

$\therefore$ Equation

$\Rightarrow$

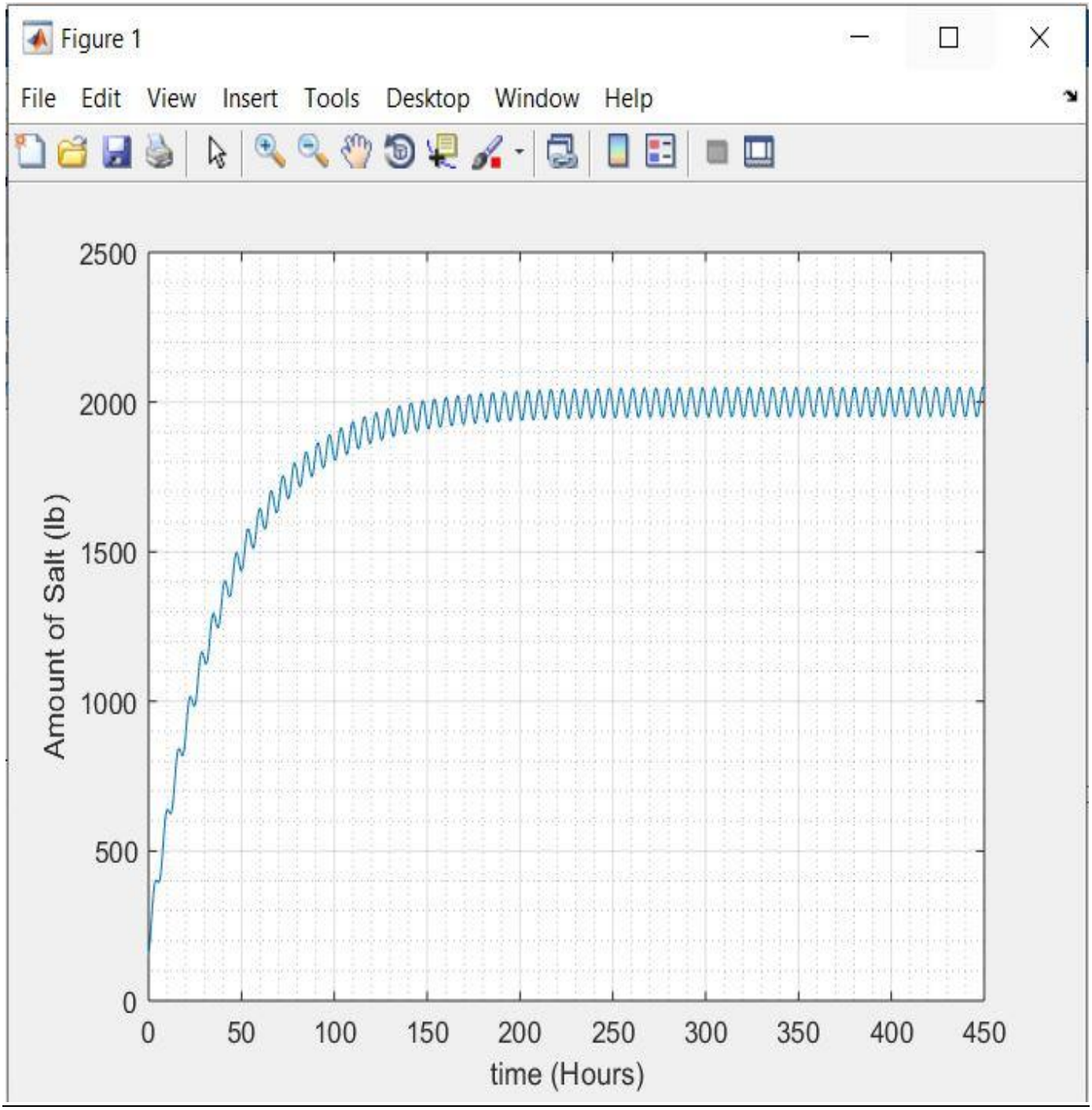
$$m = 2000\left(\left(1+\sin t\right) - \frac{1}{1.000625}\left(0.025\cos t + \sin t\right)\right) + \frac{(-1800.03)}{e^{0.025t}}$$

QUESTION 1C.

MATLAB Mfile Program to Solve the Differential Equation using "dsolve" Command.

```
commandwindow
clear
clc
close all

syms s(t)

saltin = (1 + sin(t)) * 50
saltout = (1/40)*s
saltmass = dsolve(diff(s, t) == saltin - saltout, s(0) == 150)

% 7.5 hours = 450 minutes
t = 0:0.5:450
mass = subs(saltmass, t)

plot(t, mass)
grid on
grid minor
xlabel('time (hours)')
ylabel('Amount of Salt (lb)')
```

Dynamic Response of the System for $0 \leq T \leq 7.5$ Hr with $\Delta = 0.5$min.



QUESTION 2A and B.

MATLAB mfile program to simulate the two models and insert the responses of the main dynamic model and those of the mean dynamic model in the odd-numbered and the even-numbered time values, respectively, for $0 \leq t \leq 500$ min and $\Delta = 1$min.

```
commandwindow
clear
clc
close all

syms t
values = []
t = 1:1:500
ym = 1000-((exp(-0.05*t))*800)
y = 1000 + (50/1.0025)*sin(t) + (2.5/1.0025)*cos(t) - ((exp(-0.05*t))*802.49)

if rem(t,2) == 0
    values = [values, ym]
else
    values = [values, y]
end

excelvalues = transpose(values)
mins = transpose(t)
plot(t, values)
```

```
12 -    if rem(t,2) == 0
13 -        values = [values, ym]
14 -    else
15 -        values = [values, y]
16 -    end
17
18 -    excelvalues = transpose(values)
19 -    mins = transpose(t)
20 -    plot(t, values)
21 -    grid on
22 -    grid minor
23 -    xlabel('time (minutes)')
24 -    ylabel('Volume (litres)')
25 -    xlswrite('odevbesdata.xlsx', {'t(minutes)'}, 'Veriler', 'A1')
26 -    xlswrite('odevbesdata.xlsx', mins, 'Veriler', 'A2')
27 -    xlswrite('odevbesdata.xlsx', {'Volume(litres)'}, 'Veriler', 'B1')
28 -    xlswrite('odevbesdata.xlsx', excelvalues, 'Veriler', 'B2')
29
```

QUESTION 2B

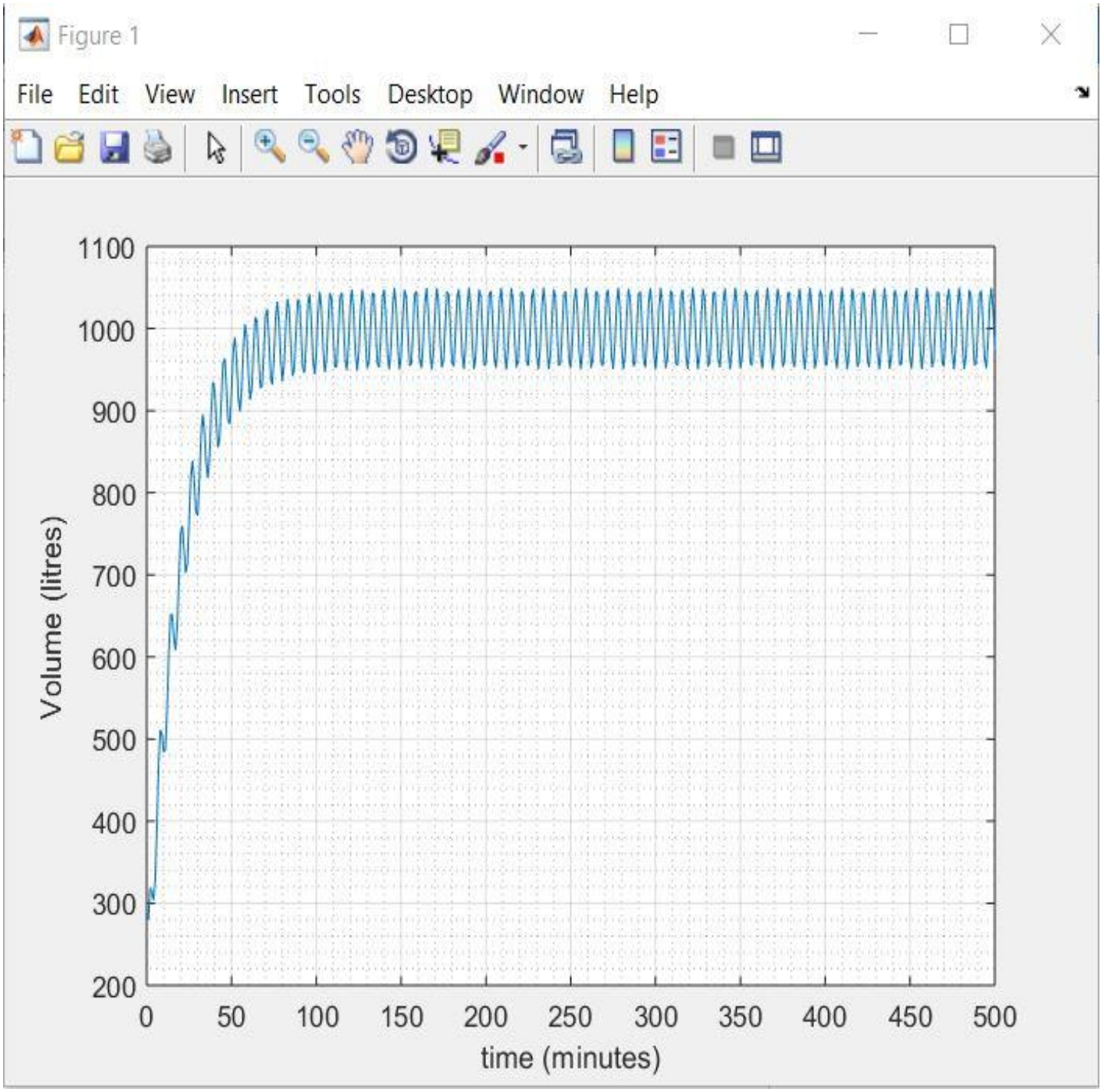
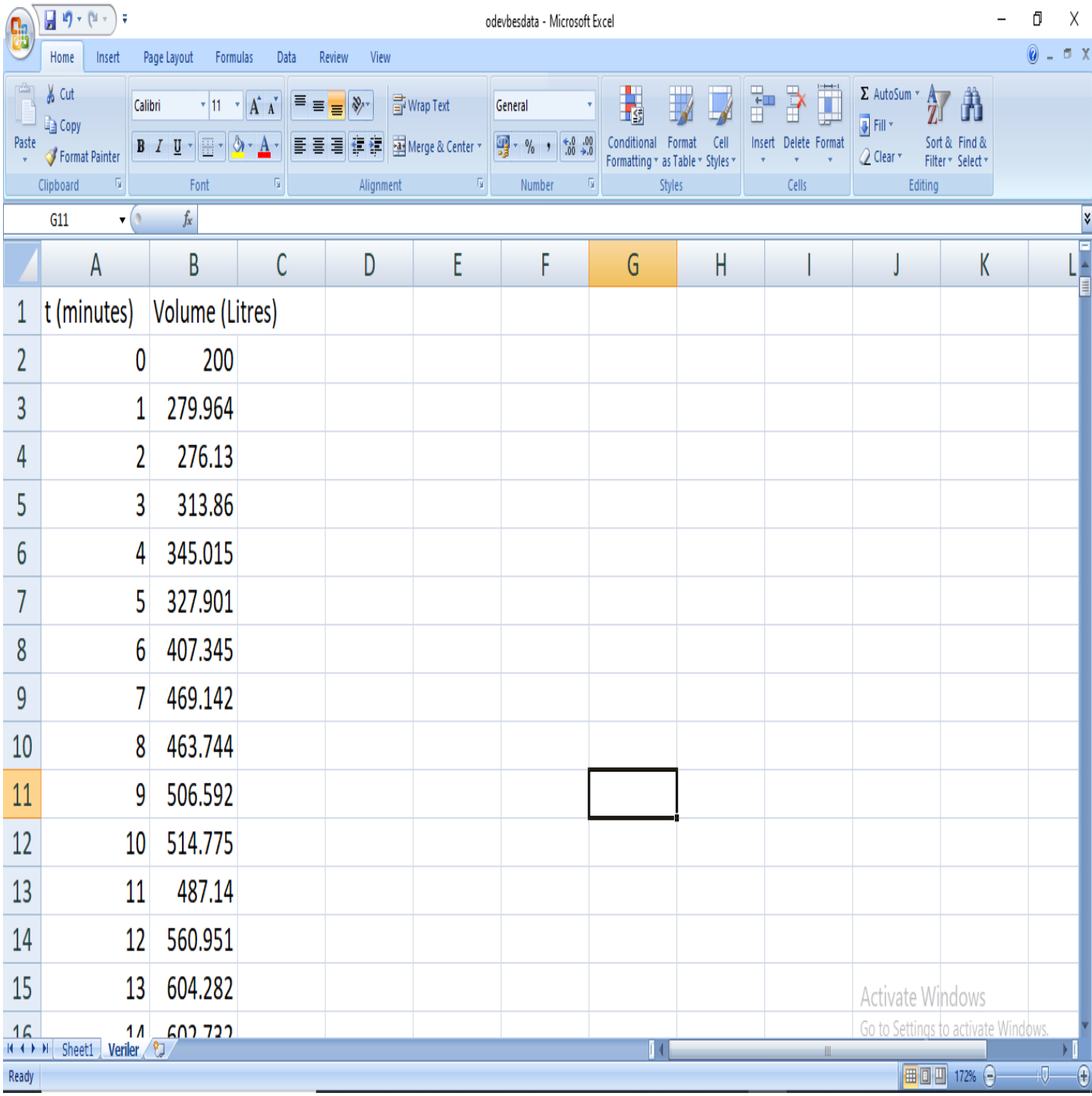
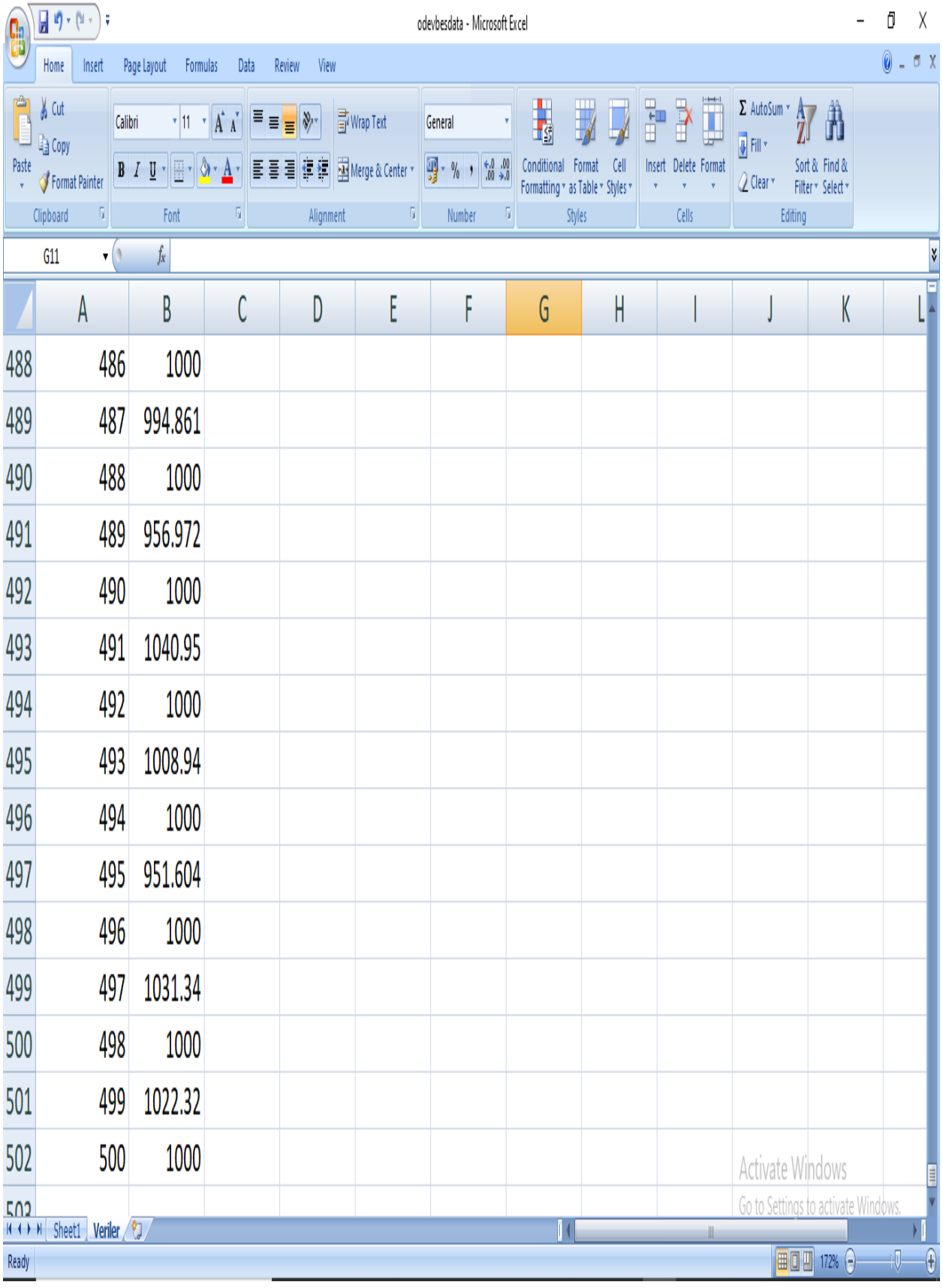


Figure 1
File Edit View Insert Tools Desktop Window Help
Volume (litres)
1100
1000
900
800
700
600
500
400
300
200
0 50 100 150 200 250 300 350 400 450 500
time (minutes)

QUESTION 2C



	A	B	C	D	E	F	G	H	I	J	K	L
1	t (minutes)	Volume (Litres)										
2	0	200										
3	1	279.964										
4	2	276.13										
5	3	313.86										
6	4	345.015										
7	5	327.901										
8	6	407.345										
9	7	469.142										
10	8	463.744										
11	9	506.592										
12	10	514.775										
13	11	487.14										
14	12	560.951										
15	13	604.282										
16	14	602.732										


odevbesdata - Microsoft Excel
Home
Insert
Page Layout
Formulas
Data
Review
View
Paste
Cut
Copy
Format Painter
Clipboard
Calibri
11
Font
Wrap Text
Merge & Center
Alignment
General
Number
Conditional Formatting
Format as Table
Cell Styles
Styles
Insert
Delete
Format
Cells
AutoSum
Fill
Clear
Sort & Filter
Find & Select
Editing
G11
A B C D E F G H I J K L
488 486 1000
489 487 994.861
490 488 1000
491 489 956.972
492 490 1000
493 491 1040.95
494 492 1000
495 493 1008.94
496 494 1000
497 495 951.604
498 496 1000
499 497 1031.34
500 498 1000
501 499 1022.32
502 500 1000
Activate Windows
Go to Settings to activate Windows.
Sheet1
Veriler
Ready
172%