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ENG 282 ASSIGNMENT

1. {Accumulation rate of salt within a system} = {Input rate of salt into the system} - {Output rate of salt from the system}

Denoting the amount of salt present in the tank at any time t as m, its time rate of change is given as

$$\frac{dm}{dt} = m_{in} - m_{out} \quad \text{—— (1)}$$

Since 50 gallons enter per minute and one gallon contains $(1+\sin t)$ lb of salt, it means that the amount of salt entering the tank is,

$$m_{in} = 50\,\frac{gal}{min} \times (1+\sin t)\,\frac{lb}{min}$$

The tank contains 1200 gallons of water with the dissolved salt, and 30 gallons of the solution leaves the tank per minute. That is,

$$\frac{30\,gal}{1200\,gal} = 0.025 = 2.5\% \text{ of the content of the tank.}$$

$$m_{out} = 2.5\% \text{ of } m$$

Therefore, from equation (1),

$$\frac{dm}{dt} = 50(1+\sin t) - 2.5\%\, m \quad \text{—— (2)}$$

$$\frac{dm}{dt} = 50(1+\sin t) - 0.025m \quad \text{—— (3)}$$

$$\frac{dm}{dt} + 0.025m = 50(1+\sin t) \quad \text{—— (4)}$$

Comparing equation 4 with the form, $\frac{dy}{dx} + Py = Q$

We have $Q = 50(1+\sin t) = (50 + 50\sin t)$

$P_y = 0.025m$

$IF = e^{\int P dt}$

$\int P dt = \int 0.025 dt$

$= 0.025t$

$IF = e^{0.025t}$

Introducing the formula

$m \cdot IF = \int Q \cdot IF \, dt$

$m \cdot e^{0.025t} = \int (50 + 50\sin t)(e^{0.025t}) dt$

Taking the RHS into consideration first

$= \int (50 + 50\sin t)(e^{0.025t}) dt$ OR $= \int (50 + 50\sin t)(e^{1/40 t})$

$= 50 \int (1 + \sin t) e^{t/40} dt$

$= 50 \int e^{t/40} (1 + \sin t) dt$

$= \int e^{t/40} (1 + \sin t) dt$

Let $u = t/40$

$t = 40u$

Let $dt = 40 du$

Let $\frac{du}{dt} = \frac{1}{40}$

$= \int e^{u} (1 + \sin 40u) 40 du$

$= 40 \int e^{u} (1 + \sin 40u) du$

Using $\int x dy = xy - \int y dx$

Let $x = 1 + \sin 40u$

$y = e^{u}$

$dx = 40\cos 40u$

$dy = e^{u}$

$(1 + \sin 40u)e^u - \int (40e^u \cos 40u)e^u du$

$\Rightarrow e^u(1 + \sin 40u) - \int 40e^u \cos 40u \, du$ ——— (6)

$e^u(1 + \sin 40u - (40\int e^u \cos 40u \, du)$

Taking $40\int e^u \cos 40u \, du$ into consideration ——— (7)

$\int x\,dy = xy - \int y\,dx$

Let $x = \cos 40u$

$dx = -40\sin 40u$

$y = e^u$

$dy = e^u$

$\therefore \int x\,dy = (\cos 40u)e^u - \int (-40\sin 40u)e^u \, du$

$= e^u \cos 40u - \int -40e^u \sin 40u \, du$

Taking $\int -40e^u \sin 40u \, du$ into consideration

$\int x\,dy = xy - \int y\,dx$

Let $x = -40\sin 40u$

$dx = -1600\cos 40u$

$y = e^u$

$dy = e^u$

$\therefore \int x\,dy = (-40\sin 40u).e^u - \int -1600\cos 40u \, e^u du$

Therefore

$e^u \cos 40u - (-40e^u \sin 40u) - \int -1600e^u \cos 40u \, du$

$e^u \cos 40u + 40e^u \sin 40u + 1600\int e^u \cos 40u$

$= \dfrac{40e^u \sin 40u + e^u \cos 40u}{1601}$

From equation 7

$$\Rightarrow \frac{40(40e^{u}\sin 40u + e^{u}\cos 40u)}{1601}$$

From equation 6

$$\Rightarrow e^{u}(1+\sin 40u) - \int 40e^{u}\cos 40u\, du$$

$$= e^{u}(1+\sin 40u) - \frac{40(40e^{u}\sin 40u + (e^{u}\cos 40u))}{1601}$$

Multiplying through by 40, we have;

$$40e^{u}(1+\sin 40u) - \frac{1600(40e^{u}\sin 40u + e^{u}\cos 40u)}{1601} \quad \text{——— (8)}$$

Remember that $u = \frac{t}{40}$

Substituting the value of u into equation 8

$$40e^{t/40}(1+\sin 40\,t/40) - \frac{1600(40e^{t/40}\sin 40\,t/40 + (e^{t/40}\cos 40\,t/40))}{1601} \longrightarrow (9)$$

Multiplying through by the 50 we left behind in equation (5)

$$2000e^{t/40}(1+\sin t) - \frac{80000(40e^{t/40}\sin t + e^{t/40}\cos t)}{1601} + c$$

$$= \frac{2000e^{t/40}(\sin t - 40\cos t + 1601)}{1601} + c$$

Given that $t = 0$ minutes and $m = 150$ lb (amount of salt in the tank

Given that $t = 0$ minute and $m = 150\,lb$

$$C = m.e^{0.025t} - \frac{2000e^{t/40}(\sin t - 40\cos t + 1601)}{1601}$$

Note: $C = m_0$

$$m_0 = 150 * e^{0.025(0)} - \frac{2000e^{0/40}(\sin 0 - 40\cos 0 + 1601)}{1601}$$

$$m_0 = 150 - \frac{(2000 * 1561)}{1601}$$

$$m_0 = 150 - 1950.03123$$

$$m_0 = -1800.03\,lb$$

From here, we can deduce that,

$$m = -1800.03e^{-t/40} + \frac{2{,}000e^{t/40}(\sin t - 40\cos t + 1601)}{1601e^{t/40}}$$

$$m = \frac{2000^{t/40}(\sin t - 40\cos t + 1601)}{1601.e^{t/40}} - \frac{1800.03}{e^{t/40}}$$

```
clear
clc
close all
syms x t
[illegible]
t(2)=0.5:450
t=[illegible](x,t)
plot(t,x)
```

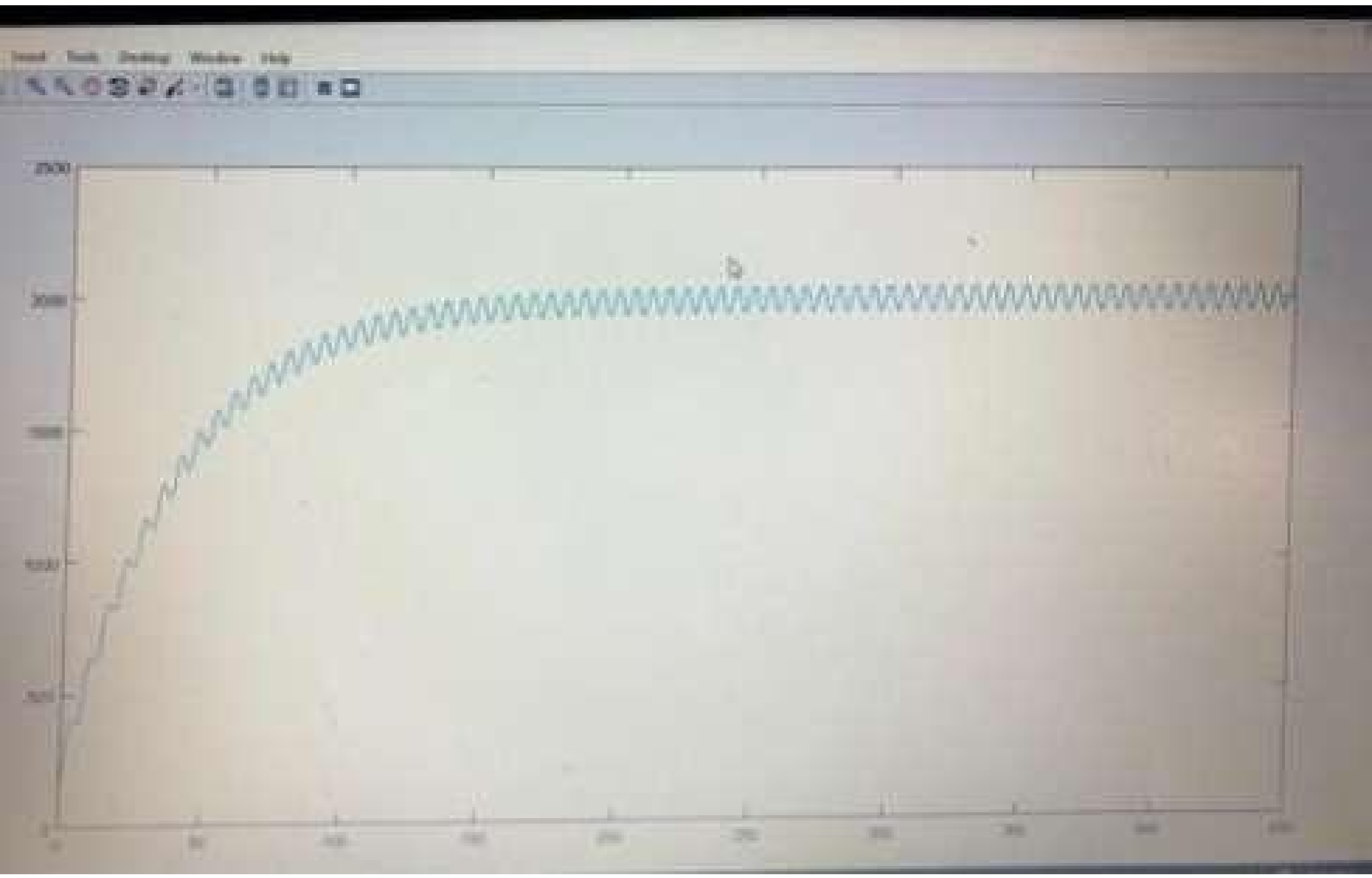
Command Window

New to MATLAB? See resources for Getting Started.

```
  443.5000  444.0000  444.5000  445.0000  445.5000  446.0000  446.5000  447.0000  447.5000  448.0000  448.5000  449.0000  449.5000  450.0000

ans =

[ 150, 2500 - (2000*1601^(1/2)*cos(atan(1/60) + 1/2))/1601, 2000 - (2
```



t(min)	v(litre)
2	279.9639
4	313.8601
6	327.9009
8	469.1423
10	506.5922
12	487.1398
14	604.2824
16	651.4694
18	608.3676
20	699.585
22	759.541
24	702.3679
26	765.9535
28	838.9333
30	776.7953
32	811.8028
34	895.7197
36	836.9388
38	843.6308
40	934.561
42	886.318
44	866.4219
46	959.1673
48	927.1558
50	883.9312
52	972.6189