NAME: OKWUOKWU BRYAN COURSE: MAT 104

MATRIC No.: 19/ENG05/049 DEPARTMENT: Mechatron

(1) $\frac{4x^2 - \sin x}{x^3} = \frac{4x^2}{x^3 x} - \frac{\sin x}{x^3}$

$\therefore \lim_{x \to 0} \frac{4}{x} - \frac{\sin x}{x^3}$ [Using L'Hopital's Rule]

$\lim_{x \to 0} \frac{\frac{d4}{dx}}{dx/dx} - \frac{d \sin x / dx}{dx^3 / dx}$

$\lim_{x \to 0} 0 + \frac{\cos x}{3x^2} \Rightarrow \frac{1}{3} \lim_{x \to 0} \frac{\cos x}{x^2}$

$\frac{1}{3} \lim_{x \to 0} \frac{\frac{d \cos x}{dx}}{d x^2 / dx} \Rightarrow \frac{1}{2 \times 3} \lim_{x \to 0} \frac{\sin x}{x}$

$= \frac{1}{6} \times 1$

$\therefore \lim_{x \to 0} \frac{4x^2 - \sin x}{x^3} = \frac{1}{6}$

(2) $\ln y = \ln (7x^2 \cos 8x / e^{3x})$

$\ln y = \ln 7x^2 + \ln \cos 8x - \ln e^{3x}$

$\ln y = \frac{14x}{7x^2} + \frac{8 \sin 8x}{\cos 8x} - \frac{3e^{3x}}{e^{3x}}$

$$\ln y = \frac{2}{x} + 8\tan 8x - 3$$

$$\frac{dy}{dx} = 2x^{-1} + 8\tan 8x - 3 \times y$$

$$= \left(2x^{-1} + 8\tan 8x - 3x\right) \times \left(\frac{7x^2 \cos 8x}{e^{3x}}\right)$$

(3) $y = \cos(5x^2 + 6x)$

$y = \cos u \qquad d(y)/du = \sin u$

$u = 5x^2 + 6x \qquad \frac{du}{dx} = 10x + 6$

$$\frac{dy}{du} \times \frac{du}{dx} = \frac{dy}{dx} = (10x + 6) \times \sin u$$

$\therefore dy/dx = (10x + 6) \times \sin(5x^2 + 6x)$

(4) (a) $3\int \frac{dx}{4x+1} = 3\int \frac{1}{4x+1}\, dx$

$u = 4x - 1 \qquad \frac{du}{dx} = 4 \qquad du = 4\,dx$

$dx = \frac{du}{4}$

$$3\int \frac{1}{u}\frac{du}{4} = \frac{3}{4}\int \frac{du}{u}$$

Recall ; $\int \frac{dx}{x} = \ln|x| + C$

$$\therefore \frac{3}{4}\int \frac{du}{u} = \frac{3}{4}\ln|u| + C$$

$$= \frac{3}{4}\ln|4x-1| + C$$

(b) $\int \frac{dx}{x^2+49} = \int \frac{dx}{x^2+7^2}$

$x = 7\tan\theta \qquad \frac{dx}{d\theta} = 7\sec^2\theta \qquad dx = 7\sec^2\theta\, d\theta$

$x^2 + 7^2 = 7^2\tan^2\theta + 7^2 = 7^2(\tan^2\theta + 1) = 49\sec^2\theta$

$$\Rightarrow \int \frac{7\sec^2\theta\, d\theta}{49\sec^2\theta\, d\theta} = \int \frac{d\theta}{7} = \frac{1}{7}\int d\theta$$

$$= \frac{1}{7}\tan^{-1}(x/7) + C$$