

Maths

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1a) From  $\frac{dy}{dt} = y \ln - y \cos t$

$$\frac{dy}{dt} = 50(1 + \sin t) - 2.5\% \text{ of } y$$

$$\frac{dy}{dt} = 50(1 + \sin t) - 0.025y$$

$\therefore$  By separating the variables

$$\frac{dy}{dt} + 0.025y = 50(1 + \sin t)$$

multiply both sides by dt

$$1 + 0.025y \, dy = 50(1 + \sin t) \, dt$$

$$1b) \frac{dy}{dt} = 50(1 + \sin t) - 0.025y$$

$$\therefore \frac{dy}{dt} + 0.025y = 50(1 + \sin t)$$

Using the linear equation method

$$\frac{dy}{dt} + P_y = Q$$

$$\therefore P = 0.025, Q = 50(1 + \sin t)$$

$$\therefore \int P \cdot dt = 0.025t$$

$$I \cdot F = e^{0.025t}$$

$$I \cdot F = e^{0.025t}$$

$$\therefore \int Q \cdot I \cdot F \cdot dt$$

$$\int e^{0.025t} = 50 \int (1 + \sin t) e^{0.025t} dt$$

$$= 50 \int e^{0.025t} + e^{0.025t} \sin t \cdot dt$$

$$= 50 \int e^{0.025t} dt + \int e^{0.025t} \sin t \cdot dt$$

$$= 50 \cdot \frac{e^{0.025t}}{0.025} + \int e^{0.025t} \sin t \cdot dt$$

Using integration by part  $\int u \cdot dv = uv - \int v \cdot du$

$$\int e^{0.025t} \sin t \cdot dt$$

$$u = e^{0.025t} \quad dv = \sin t$$

$$du = 0.025 e^{0.025t} \quad v = -\cos t$$

$$\therefore \int e^{0.025t} \sin t = e^{0.025t} \cdot -\cos t - \int -\cos t \cdot 0.025 e^{0.025t}$$

$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t - \int -\cos t \cdot 0.025 e^{0.025t} dt$$

~~$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t - \int -\cos t \cdot 0.025 e^{0.025t}$$~~

$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t + 0.25 \int e^{0.025t} \cos t + c$$

Using Integration by part,

$$\int u dv = uv - \int v du$$

$$u = e^{0.025t}, dv = \cos t$$

$$du = 0.025 e^{0.025t}, v = \sin t$$

$$= -e^{0.025t} \cos t + 0.025 \left[ e^{0.025t} \sin t - \int \sin t \cdot 0.025 e^{0.025t} \right]$$

$$= -e^{0.025t} \cos t + 0.025 \left[ e^{0.025t} \sin t - 0.025 \int \sin t e^{0.025t} \right]$$

$$\text{Let } Q = \int e^{0.025t} \sin t$$

$$\therefore Q = -e^{0.025t} \cos t + 0.025 \int e^{0.025t} \sin t - 0.025 Q$$

$$Q = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t - 6.25 \cdot 10^{-4} Q$$

$$Q + 6.25 \cdot 10^{-4} Q = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t$$

$$Q + 0.000625 Q = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t$$

$$1.000625 Q = -e^{0.025t} \cos t + 0.025 e^{0.025t} \sin t$$

$$1.000625 Q = -e^{0.025t} (\cos t - 0.025 \sin t)$$

$$Q = \frac{-e^{0.025t} (\cos t - 0.025 \sin t) + c}{1.000625}$$

$$y = 2000 - \frac{50}{1.000625} (\cos t - 0.025 \sin t) + \frac{50c}{e^{0.025}}$$

When  $y = 150$

$$t = 0$$

$$150 = 2000 - \frac{50}{1.000625} (1 - 0) + \frac{50c}{1}$$

$$Q = \frac{-e^{0.025t}}{1.000625} (\cos t - 0.025) + c$$

$$\int e^{0.025t} \sin t = \frac{-e^{0.025t}}{1.000625} (\cos t - 0.025) + c$$

$$\text{Since } \int e^{0.025t} \sin t = \frac{-e^{0.025t}}{1.000625} (\cos t - 0.025) + c$$

$$\therefore y e^{0.025t} = 50 \left[ \frac{e^{0.025t}}{0.025} - \frac{e^{0.025t}}{1.000625} (\cos t - 0.025) + c \right]$$

$$y e^{0.025t} = 2000 e^{0.025t} - 50 \cdot \frac{e^{0.025t}}{1.000625} (\cos t - 0.025) + 50c$$

$$y = 2000 - \frac{50}{1.000625} (\cos t - 0.025) + \frac{50c}{e^{0.025t}}$$

$$150 = 2000 - 49.968(1) + 50c$$

$$150 = 1950.032 + 50c$$

$$-1800.032 = 50c$$

$$c = -36.00064$$

commandwindow

```
1 - clear
2 -
3 - clc
4 - close all
5 - syms m t
6 - ans=dsolve('Dm+0.025*m=50+50*sin(t)', 'm(0)=150')
7 - t=0:0.5:450
8 - tn=subs(ans,t)
9 - plot(t,tn)
```

I

Command Window

New to MATLAB? See resources for [Getting Started](#).

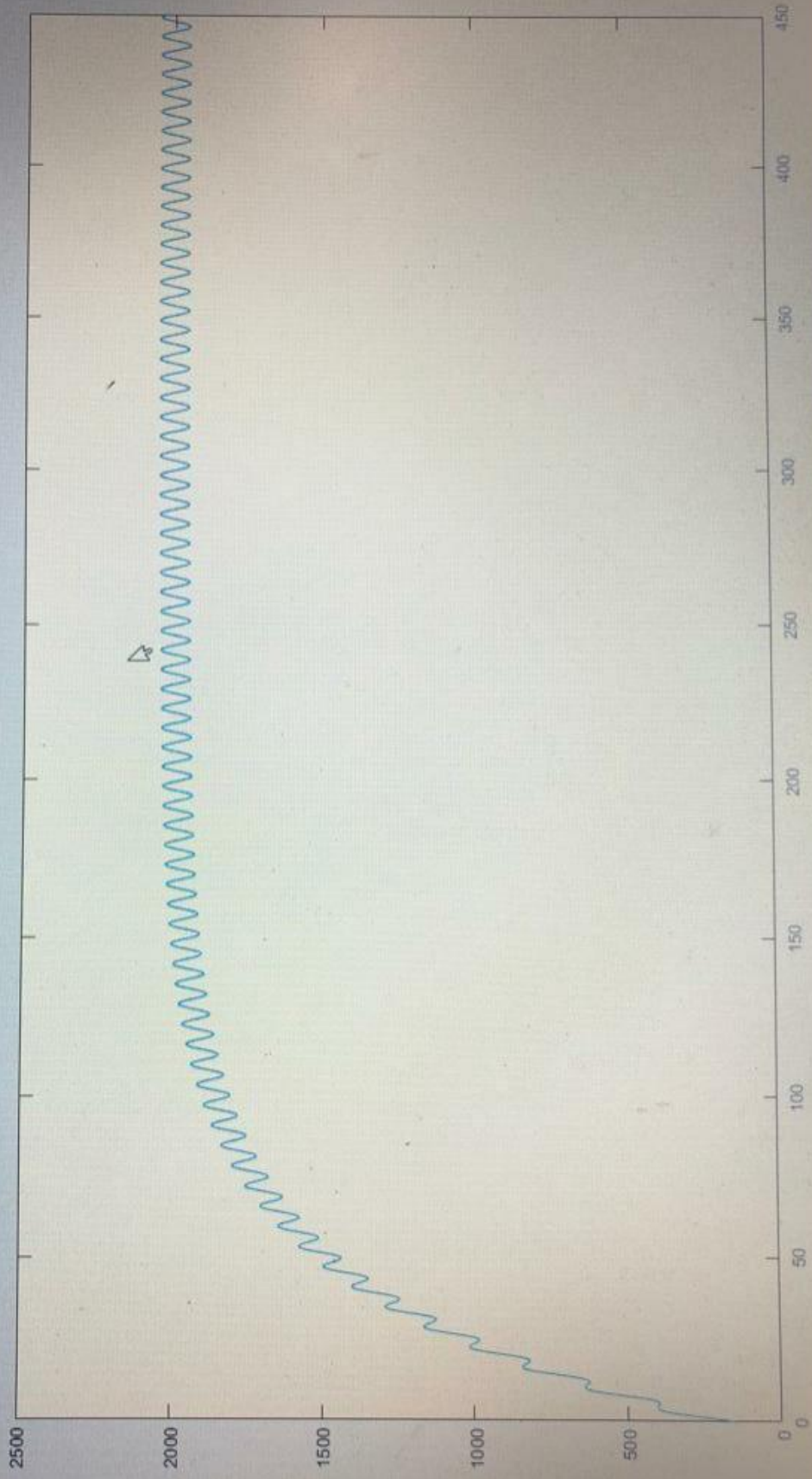
```
445.5000 446.0000 446.5000 447.0000 447.5000 448.0000 448.5000 449.0000 449.5000 450.0000
```

tn =

```
[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, ...
```

fx >>

script



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
82																					
1	t(min)	V(Litre)																			
2	1	279.9639																			
3	2	318.1907																			
4	3	313.8601																			
5	4	303.601																			
6	5	327.9009																			
7	6	393.9593																			
8	7	469.1423																			
9	8	511.0566																			
10	9	506.5922																			
11	10	484.0395																			
12	11	487.1398																			
13	12	534.9268																			
14	13	604.2824																			
15	14	651.2431																			
16	15	651.4694																			
17	16	622.6706																			
18	17	608.3676																			
19	18	637.9229																			
20	19	699.585																			
21	20	751.3315																			
22	21	759.541																			
23	22	729.9392																			
24	23	702.3679																			
25	24	714.1865																			
26	25	765.9535																			
27	26	820.9421																			
28	27	838.9333																			
29	28	813.2194																			
30	29	776.7953																			