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1) Using "balance law", The acceleration rate of Salt within a System is equal to the input rate of the salt into the System minus the output rate of the salt from the System.

Acceleration rate of Salt within a System

$$= \text{Input rate of Salt into the System} - \text{Output rate of Salt from the System}$$

Let the amount of Salt present in the tank at any time "t" be "y".

$$\text{Time rate of change of } y = \frac{dy}{dt} = y_{in} - y_{out}$$

If 50 gal enters the tank per minute and one gallon contains $(1 + 50 \times 10^{-4}) = (1 + 5 \times 10^{-3}) = 1.025$ lb

Hence the amount of salt entering into the tank is

$$50 \text{ gal/min} \times 1.025 \text{ lb} = 51.25 \text{ lb/min}$$

The tank contains 1200 gal of water with dissolved salt and 30 gal of the solution exist the tank per min i.e. $\frac{30 \text{ gal}}{1200 \text{ gal}} = 0.025$

= 2.5% of the content of the tank

$$= 2.5\%$$

So 2.5% of the salt present inside the tank will also leave the tank per minute. i.e.

$$y_{out} = 2.5\% \text{ of } y$$

$$a) \frac{dy}{dt} = 51.25 \text{ lb/min} - 2.5\% \text{ of } y \text{ lb/min}$$

$$b) \frac{dy}{dt} = 51 - 0.025y; \quad \frac{dy}{dt} = -0.025y + 51$$

$$\frac{dy}{dt} = -0.025 \left(\frac{-0.025y + 51}{-0.025} \right)$$

$$\frac{dy}{dt} = -0.02 (y - 2040)$$

$$\left(\frac{dy}{y - 2040} \right) = -0.025 dt; \quad \int \frac{dy}{(y - 2040)} = \int -0.025 dt$$

$$\int \frac{dy}{y - 2040} = -0.025 \int dt; \quad \ln(y - 2040) = -0.025t + C$$

$$y - 2040 = e^{-0.025t + C}$$

$$y - 2040 = e^{-0.025t} \cdot e^C$$

$$y - 2040 = e^{-0.025t} y_0; \quad y - 2040 = y_0 e^{-0.025t}$$

$$y = y_0 e^{-0.025t}$$

$$+ 2040; \quad \text{Remember when } t = 1, y = 150$$

$$150 - 150 = y_0 e^{-0.025(1)} + 2040; \quad 150 - 2040 = y_0 \times (-1)$$

$$y_0 = -1890$$

So;

$$y = -1890 e^{-0.025t} + 2040$$

$$y = 2040 - 1890 e^{-0.025t}$$