

MATRIC NO: 17/MHS01/314

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DEPARTMENT: Mechanical Engineering

COURSE: Engineering Mathematics.

1. Given that,

tank contains 120 gal of  $H_2O$  & 150 lb of salt initially dissolved  
50 gal of brine, each gallon containing  $(1 + 8 \sin t)$  lb of dissolved salt runs  
into tank per minute  
mixture leaving tank at 30 gal/minute

Considering,

$$- \frac{dm}{dt} = M_{in} - M_{out}$$

$$M_{in} = 50 \frac{\text{gal}}{\text{min}} \times (1 + 8 \sin t) \frac{\text{lb}}{\text{gal}}$$
$$= 50(1 + 8 \sin t) \frac{\text{lb}}{\text{min}}$$

$$M_{out} = \frac{30}{120} = \frac{1}{40} \text{ of } m$$

$$\Rightarrow \frac{dm}{dt} = 50(1 + 8 \sin t) - \frac{1}{40} m$$

$$\frac{dm}{dt} + \frac{1}{40} m = 50(1 + 8 \sin t)$$

Comparing to  $\frac{dy}{dx} + P_y = Q$

$$IF = e^{\int P dt}$$
$$\Rightarrow IF = e^{\int \frac{1}{40} dt}$$
$$\therefore IF = e^{\frac{1}{40} t}$$

to solve compare with,  $y \times IF = \int (Q \times IF) dx + c$

$$\Rightarrow m \cdot e^{\frac{1}{40}t} = \int 50(1 + \sin t) (e^{\frac{1}{40}t}) dt + c$$

$$= 50 \left[ \int e^{\frac{1}{40}t} + \int \sin t \cdot e^{\frac{1}{40}t} \right] \cdot dt$$

Let  $N = \int (\sin t) (e^{\frac{1}{40}t}) \cdot dt$

Compare with using,  $u = \sin t$ ;  $dv = e^{\frac{1}{40}t}$

$$\int u dv = uv - \int v du$$

$$\Rightarrow \int (\sin t) (e^{\frac{1}{40}t}) \cdot dt = (\sin t) (40 e^{\frac{1}{40}t}) - \int (40 e^{\frac{1}{40}t}) (\cos t)$$

$$N = (40 e^{\frac{1}{40}t}) (\sin t) - 40 \left[ \int (e^{\frac{1}{40}t}) (\cos t) \cdot dt \right]$$

$$N = (40 e^{\frac{1}{40}t}) (\sin t) - 40 \left[ (40 e^{\frac{1}{40}t}) (\cos t) + (e^{\frac{1}{40}t}) (\sin t) \right]$$

$$N = (40 e^{\frac{1}{40}t}) (\sin t) - 40 \left[ (\cos t) (40 e^{\frac{1}{40}t}) \right] - 1600 N$$

$$N + 1600N = (\sin t) (40 e^{\frac{1}{40}t}) - 1600 \cdot e^{\frac{1}{40}t} \cdot \cos t$$

$$1601N = (\sin t) (40 e^{\frac{1}{40}t}) - 1600 \cdot e^{\frac{1}{40}t} \cdot \cos t$$

$$50(N) \times e^{\frac{1}{40}t} = 50 \left[ (\sin t) (40 e^{\frac{1}{40}t}) - 1600 \cdot e^{\frac{1}{40}t} \cdot \cos t \right]$$

$$\therefore 50N = 50(\sin t) (40 e^{\frac{1}{40}t}) - 80,000 \cdot e^{\frac{1}{40}t} \cdot \cos t$$

But  $m \cdot e^{\frac{1}{40}t} = 50(40 e^{\frac{1}{40}t}) + 50N + c$

$$m \cdot e^{\frac{1}{40}t} = 2000 e^{\frac{1}{40}t} + 50N + c$$

$$m = 2000 + \frac{50N}{e^{\frac{1}{40}t}} + \frac{c}{e^{\frac{1}{40}t}}$$

$$m = 2000 + \frac{2000(\sin t)}{1601} - \frac{80,000(\cos t)}{1601} + \frac{c}{e^{\frac{1}{40}t}}$$

At  $t=0$ ,  $m=150$

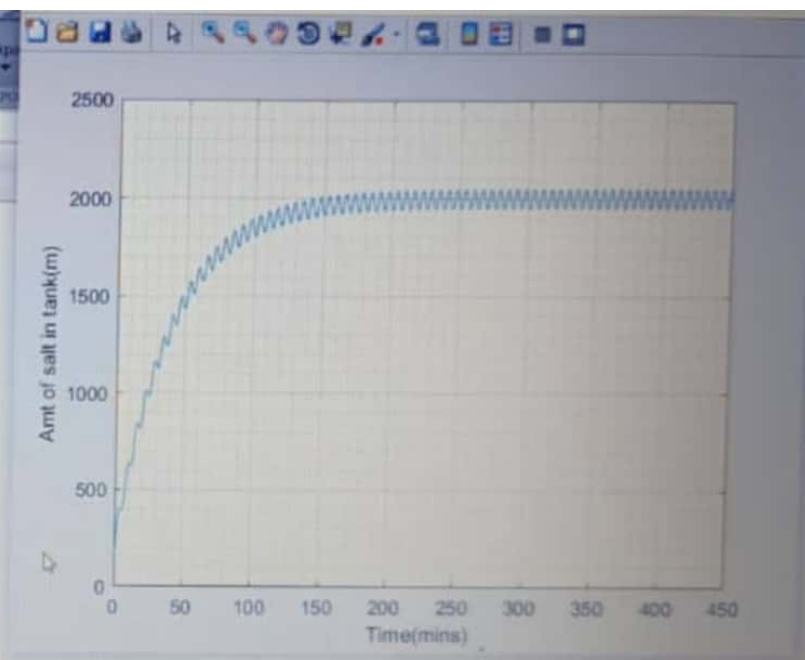
$$\Rightarrow 150 = 2000 + \frac{2000(0)}{1601} - \frac{80,000(1)}{1601} + \frac{c}{1}$$

$$\therefore c = -1800.03$$

Thus the ODE,

$$M = 2000 + \frac{9000(\sin t)}{1601} - \frac{80000(\cos t)}{1601} - \frac{1800.03}{e^{(40 \cdot t)}}$$

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1 - commandwindow
2 - clearvars
3 - clc
4 - close all
5 - syms m t
6 - And=dsolve('Dm+0.025*m=50+50*sin(t)', 'm(0)=150')
7 - tn=0:0.5:450
8 - tn=subs(And,t)
9 - plot(t,tn)
10 - xlabel('Time (mins)')
11 - ylabel('Amt of salt in tank(m)')
12 - grid on
13 - grid minor
14 - clear
15 - clc
```



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1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - syms t
6 - tn=0:1:500
7 - values=[]
8 - mean=1000-(exp(0.05*t))*800)
9 - y=1000+(50/1.0025)*sin(t)+(2.5/1.0025)*cos(t)-(exp(-0.05*t))*802.49)
10
11 - if rem(tn,2) ==0
12 -     values=[values,mean]
13 - else
14 -     values=[values,y]
15 - end
16 - excelvalues=transpose(values)
17 - mins=transpose(tn)
18 - plot(tn,values)
19 - grid on
20 - grid minor
21 - xlabel('time (mins)')
22 - ylabel('volume (litre)')

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