Question 1

Arogunmati Oluwadamilola Alexander
Mechatronics Engineering
18/ENG05/011

ENG 282 Assignment 5

(a)

Tank holds 1200gal of water with 150lb of salt dissolved in it

- Input rate => 50gal of brine/min [1 gall has (1 + sint) lb of salt]
- Output Rate => 30gal of brine/min

m => Amount of salt at any time t

Applying the balance law

[accumulation rate of salt within a system] = [input rate of salt into the system] − [output rate of salt from the system]

$$\frac{dm}{dt} = m_{in} - m_{out}$$

(i) 50gal of water enter per minute and a gallon has (1+Sint) lb of salt

$\therefore \quad m_{in} = 50\text{gal/min} \times (1+\sin t)\,\text{lb/min} = 50(1+\sin t)\,\text{lb/min}$

(ii) If the tank contains 1200gal of water and 30gal of water leaves per minute. That means $\frac{30}{1200} = \frac{1}{40} = 0.025$

=> 2.5% of the water in the tank leaves per minute
Therefore, 2.5% of the salt will also leave the tank per minute;

$m_{out} \Rightarrow 2.5\%$ of m

$$\frac{dm}{dt} = 50(1+\sin t) - 0.025m$$

— The ordinary differential equation (ODE)

18/ENG05/011

(b) $\frac{dm}{dt} = 50(\sin t + 1) - 0.025m$ — (1)

solving the differential equation using the integrating factor method

~~Comparing (1) with~~

$\frac{dm}{dt} + 0.025m = 50(\sin t + 1)$ — (2)

comparing eq (2) with $\frac{dy}{dx} + Py = Q$ $\left(\frac{dm}{dt} + Pm = Q\right)$

$P = 0.025$, $Q = 50(\sin t + 1)$

$$\boxed{e^{\int P dt} = \text{Integrating factor}}$$

$\int P dt \Rightarrow \int 0.025 dt \Rightarrow 0.025t$

$IF \Rightarrow e^{0.025t}$

from $y \cdot IF = \int Q \cdot IF dx$

$m \cdot IF = \int Q \cdot IF dt$

$m \cdot e^{0.025t} = \int 50(\sin t + 1) \cdot e^{0.025t} dt$

$\Rightarrow \int 50(\sin t + 1) \cdot e^{0.025t} dt = \int 50 e^{t/40} \cdot (\sin t + 1) dt$

$\Rightarrow 50 \int e^{t/40} (\sin t + 1) dt$

$\int e^{t/40}(\sin t + 1) dt$

$\Rightarrow$ let $u = t/40 \rightarrow du/dt = \frac{1}{40}$, $dt = 40 du$.

$\Rightarrow 40 \int e^{u}(\sin 40u + 1) du$

$\Rightarrow$ removing the constant for now

$\int e^u(\sin 40u + 1)\,du$

let $g = \sin 40u + 1$, $dv = e^u du$

$\frac{dg}{du} = 40\cos 40u$, $\int dv = \int e^u du$

$v = e^u$

$dg = 40(\cos 40u)du$

from $\int g\,dv = gv - \int v\,dg$

$\int e^u(\sin 40u+1)\,du \Rightarrow e^u(\sin 40u + 1) - \int 40e^u(\cos 40u)du$ $\longrightarrow$ ③

$\int 40e^u(\cos 40u)du \Rightarrow 40\int e^u(\cos 40u)du$

$\int e^u\cos 40u\,du \Rightarrow$ let $g = \cos 40u$ $dv = e^u du$

$\int dv = \int e^u du$

$\frac{dg}{du} = -40\sin 40u$, $v = e^u$.

$\Rightarrow \int g\,dv = gv - \int v\,dg$

$\int e^u\cos 40u\,du = e^u\cos 40u - \int -40e^u(\sin 40u)\,du$

Integrating by part again

$\int g\,dv = gv - \int v\,dg$ $g =$

$g \Rightarrow -40\sin 40u$ $dv = e^u$

$dg = -1600(\cos 40u)du$, $v = \cancel{e^u}\; e^u$.

$\int e^u\cos 40u\,du = e^u\cos 40u - (-40e^u\sin 40u - \int -1600e^u\cos 40u\,du$

$\int e^u\cos 40u\,du \Rightarrow e^u\cos(40u) - (-40e^u\sin(40u)) + 1600\int e^u\cos 40u\,du$

$\int e^u\cos 40u\,du + 1600\int e^u\cos 40u\,du = e^u\cos 40u + 40e^u\sin 40u$

$1601\int e^u\cos 40u\,du = e^u\cos 40 + 40e^u\sin 40u$

Dividing through by 1601

$$\frac{\int e^{u}(\cos 40u)\,du}{\cancel{1601}} = \frac{40e^{u}\sin 40u + e^{u}\cos 40u}{1601}$$

Bringing back the 3rd constants that was removed

$$40\int e^{u}\cos 40u\,du = \frac{40(40e^{u}\sin 40u + e^{u}\cos 40u)}{1601} \quad \text{— (4)}$$

But from eqn (3)

$$\Rightarrow \int e^{u}(\sin 40u+1)\,du = e^{u}\sin 40u+1 - \int 40e^{u}\cos 40u\,du$$

$$\int e^{u}(\sin 40u+1)\,du = e^{u}\sin 40u+1 - \frac{40(40e^{u}\sin 40u+e^{u}\cos 40u)}{1601}$$

(substituted eqn 4 in 1)

Bringing back 2nd constant removed (40)

$$40\int e^{u}\sin 40u+1\,du = 40e^{u}(\sin 40u+1) - \frac{1600(40e^{u}\sin 40u+e^{u}\cos 40u)}{1601}$$

substituting $u = t/40$ back

$$\Rightarrow 40e^{t/40}(\sin t+1) - \frac{1600(40e^{t/40} + e^{t/40}\cos(t))}{1601}$$

Bringing back the first constant removed (50)

$$50\int e^{t/40}(\sin(t)+1)\,dt$$

multiplying through by 50

$$\Rightarrow 2000e^{t/40}(\sin t+1) - \frac{80000(40e^{t/40}\sin t + e^{t/40}\cos t)}{1601}$$

$$50\int e^{t/40}(\sin t+1)\,dt$$

$$\Rightarrow \frac{2000e^{t/40}(\sin t - 40\cos t + 1601)}{1601} + c \quad \text{— (5)}$$

Going back to the original Integrating factor equation

$$m \cdot IF = \int Q \cdot IF \, dt$$

$$m \cdot e^{0.025t} = \int 50(\sin t + 1) \cdot e^{0.025t} \, dt$$

substitution eqn (5)

$$m \cdot e^{0.025t} = \frac{2000 e^{0.025t}(\sin t - 40\cos t + 1601)}{1601} + c$$

$$m = \frac{2000 \cancel{e^{0.025t}}(\sin t - 40\cos t + 1601)}{1601 \cdot \cancel{e^{0.025t}}} + \frac{c}{e^{0.025t}}$$

$$m = \frac{2000(\sin t - 40\cos t + 1601)}{1601} + m_0 \cdot e^{-0.025t} \quad \text{--- (6)}$$

at $t = 0$ min and $m = 150$ lb of salt

$$150 = \frac{2000(\sin 0 - 40\cos(0) + 1601)}{1601} + m_0 \cdot 1$$

$$150 = \frac{2000(-40 + 1601)}{1601} + m_0$$

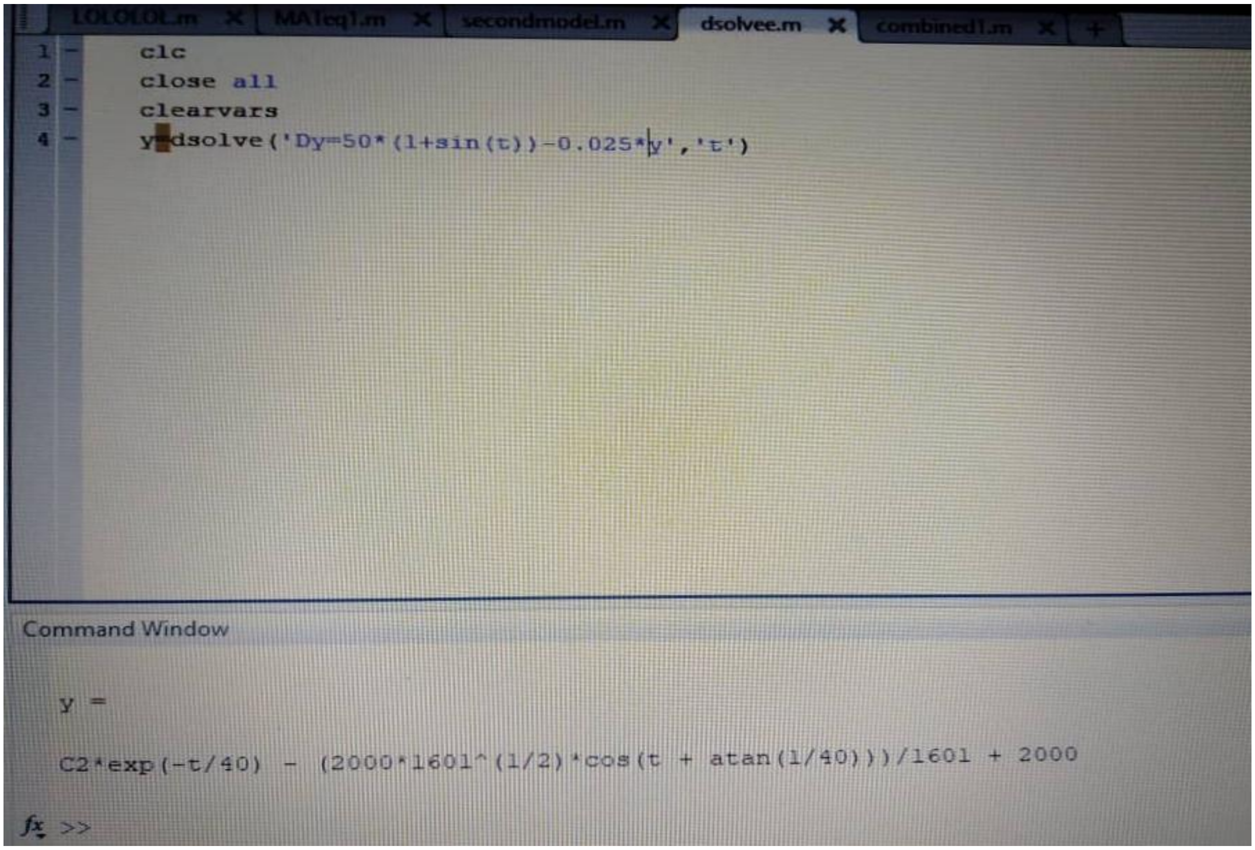
$$150 = 1950.03 + m_0$$

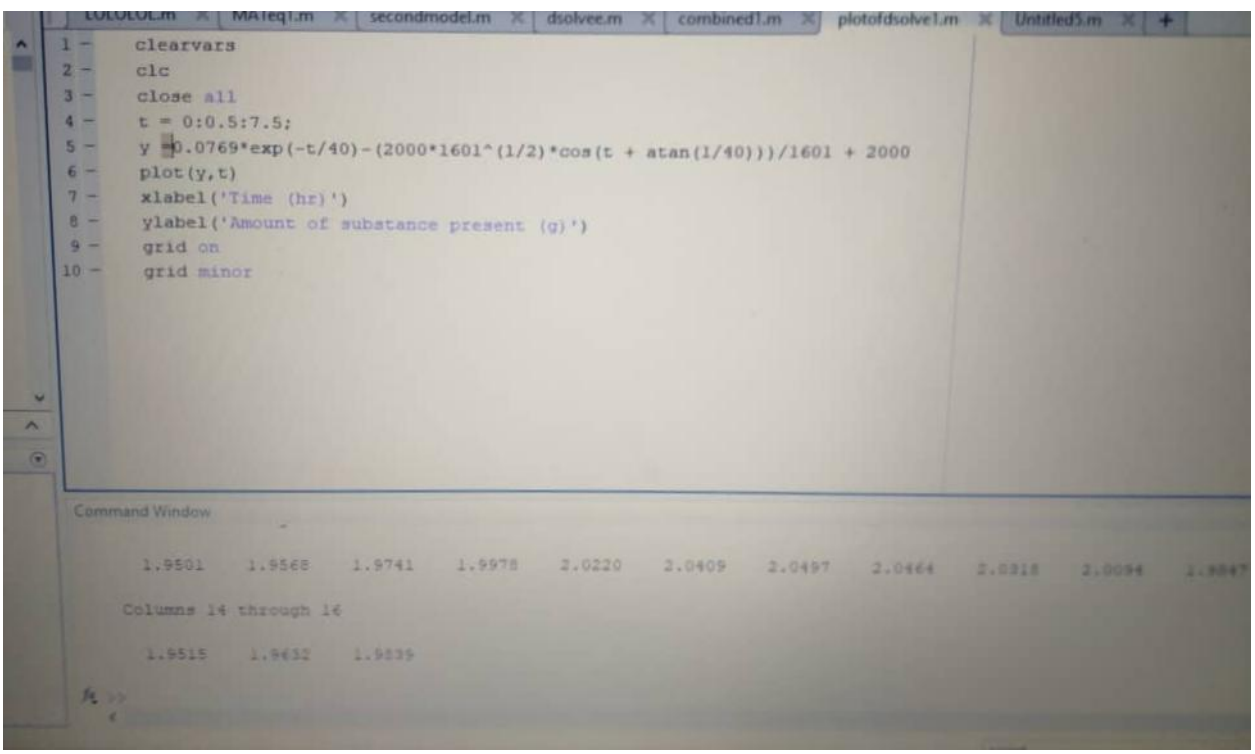
$$m_0 = 150 - 1950.03 = -1800.03$$

substituting m_0 into eqn (6)

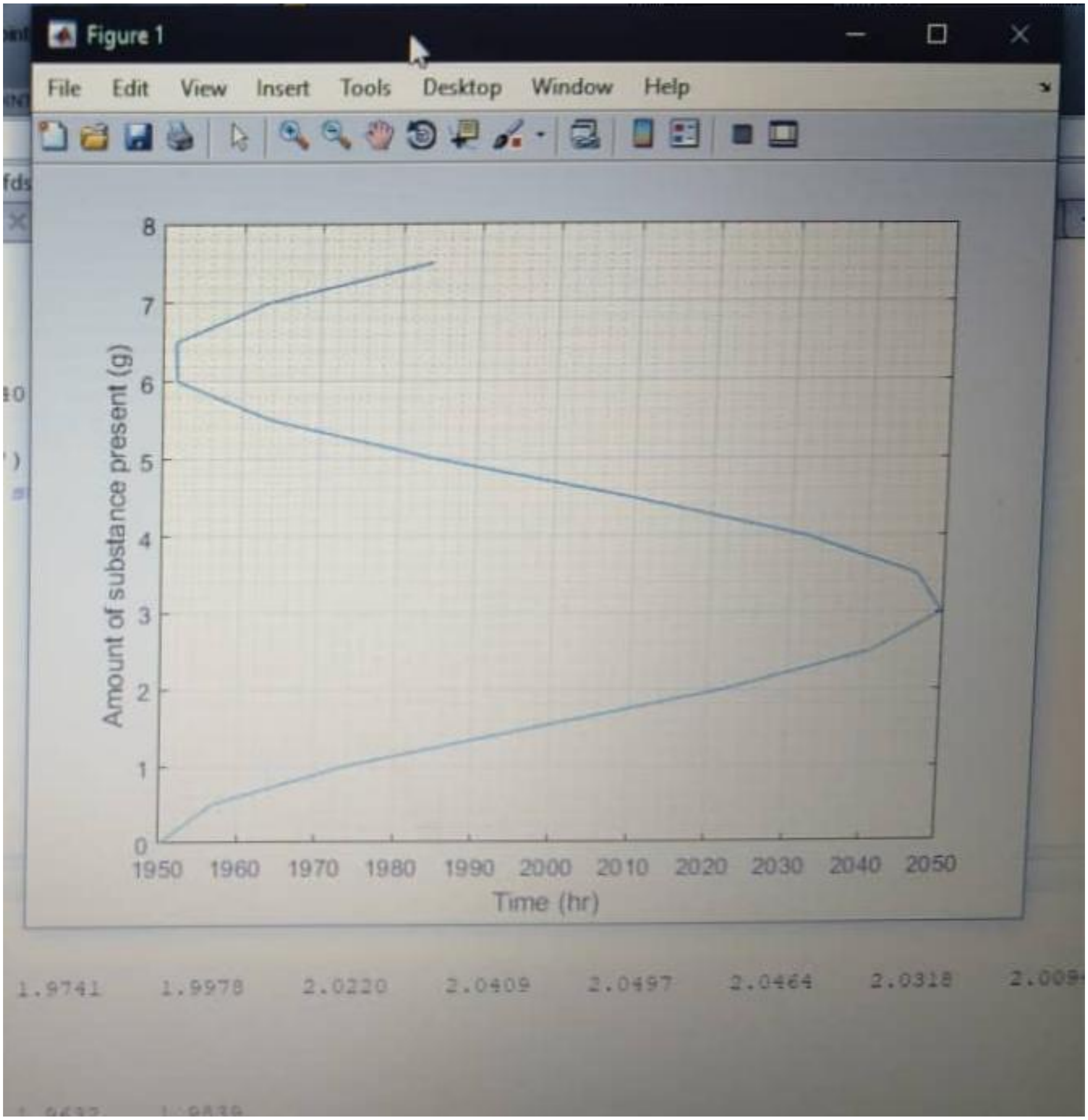
$$m = \frac{2000(\sin t - 40\cos t + 1601)}{1601} - \frac{1800.03}{e^{0.025t}}$$

Solution to Differential Equation using matlab



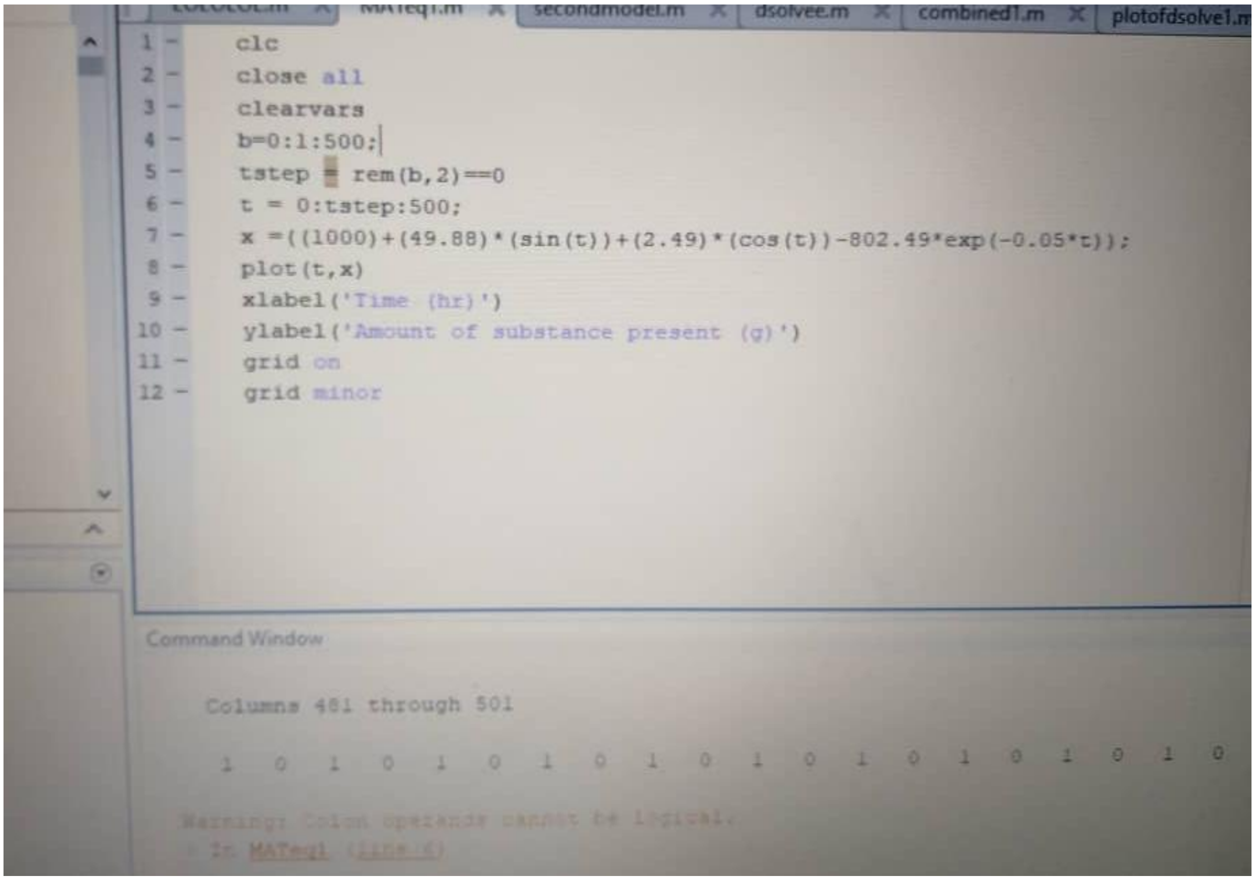


Dynamic Response

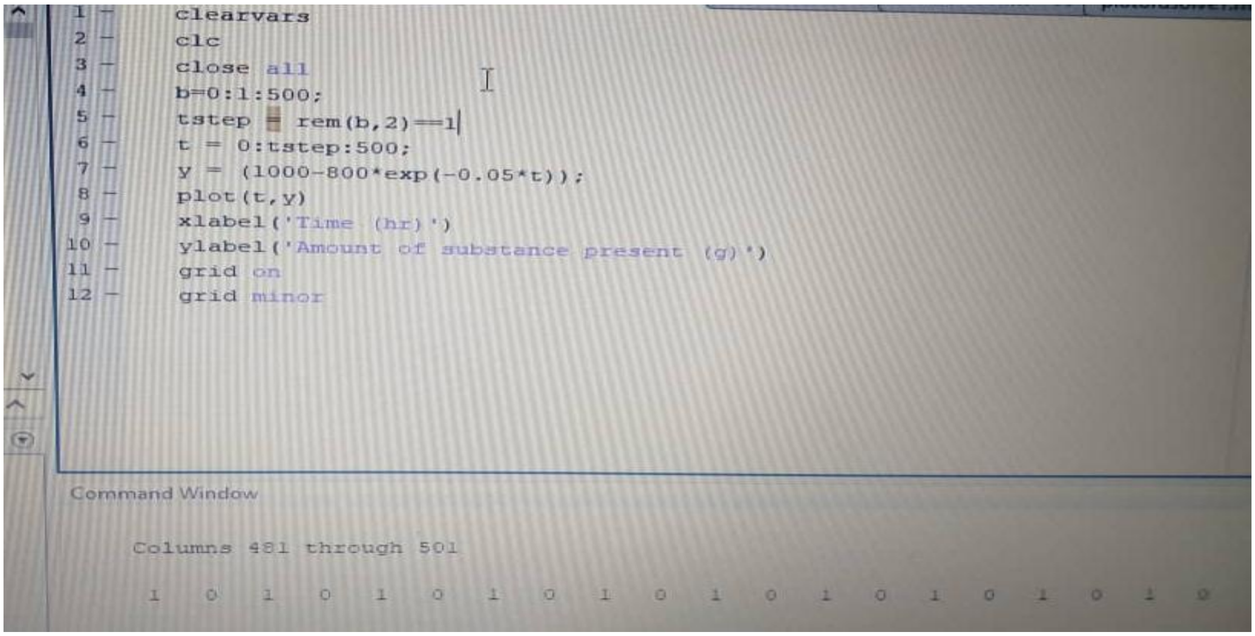


Question 2

Model 1



Model 2



```
clearvars
clc
close all
b=0:1:500;
tstep = rem(b,2)==1
t = 0:tstep:500;
y = (1000-800*exp(-0.05*t));
plot(t,y)
xlabel('Time (hr)')
ylabel('Amount of substance present (g)')
grid on
grid minor
```

Command Window

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Columns 481 through 501

1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
```

<u>Dynamic Response</u>