Name | Bawwann Khashmn Bule
Dept | Civil Engineering
Mat No | 18/ENG03/022

(1a) The rate of change is given as $dy/dt = y_{in} - y_{out}$
and at $t = 1'; 50(1+\sin t)$ lb of salt is in one gallon
$\therefore 50(1+\sin(1))$ lb ~~= 1.02lb~~ of salt
$\therefore y_{in} = 50\ gal/min \times 1.02\ lb/gal = 51\ lb/min$
Since the tank has 1200 gal of water dissolved with salt and 30 gallons of the solution leave per minute it becomes $30/1200\ gal = 0.025$
$= 2.5\%$ of the brine in the ~~tank~~ tank
if the same amount exits the tank it becomes
$y_{out} = 0.025y$ or 2.5% of y
$\therefore dy/dt = 50(1+\sin t) - 0.025y$

As a differential equation it becomes
$$dy/dt + 0.025y = 50(1+\sin t)$$

(1b) Using the linear equation method
$$dy/dx + Py = Q$$
where $P = 0.025$, $Q = 50(1+\sin t)$
$\int P.dt = 0.025t$
$I.F = e^{\int P.dt}$
$I.F = e^{0.025t}$
$y * I.F = \int Q\, I.F.\, dt$
which is
$$ye^{0.025t} = \int 50(1+\sin t)\, e^{0.025t}\, dt$$

$$ye^{0.025t} = 50\int e^{0.025t} + e^{0.025t}\sin t \cdot dt$$

$$ye^{0.025t} = 50 \cdot \frac{e^{0.025t}}{0.025} + \int e^{0.025t}\sin t \cdot dt$$

Using integration by part, $\int u\,dv = uv - \int v\,du$

$$\int e^{0.025t}\sin t \cdot dt$$

$u = e^{0.025t} \quad dv = \sin t$

$du = 0.025e^{0.025t} \quad v = -\cos t$

$$\therefore \int e^{0.025t}\sin t = e^{0.025t} \cdot -\cos t - \int -\cos t \cdot 0.025e^{0.025t}$$

Let $Q = \int e^{0.025t}\sin t$

$$Q = -e^{0.025t}\cos t + 0.025\int e^{0.025}\sin t - 0.025Q$$

$$Q + 6.25^{-4}Q = -e^{0.025t}\cos t + 0.025e^{0.025t}$$

$$Q + 0.000625Q = -e^{0.025t}\cos t + 0.025e^{0.025t}$$

$$1.000625Q = -e^{0.025t}\cos t + 0.025e^{0.025t}$$

$$1.000625Q = -e^{0.025t}(\cos t - 0.025)$$

$$Q = \frac{-e^{0.025t}(\cos t - 0.025)}{1.000625} + C$$

Recall

$$Q = \int e^{0.025t}\sin t$$

$$\int e^{0.025t}\sin t = \frac{-e^{0.025t}(\cos t - 0.025)}{1.000625} + C$$

Since $\int e^{0.025t}\sin t = \frac{-e^{0.025t}(\cos t - 0.025)}{1.00625} + C$

$$ye^{0.025t} = 50\left[\frac{e^{0.025t}}{0.025} - \frac{e^{0.025t}(\cos t - 0.025)}{1.000625}\right]$$

$$ye^{0.025t} = 2000e^{0.025t} - \frac{50e^{0.025t}(\cos t - 0.025)}{1.000625} + 50c$$

$$y = 2000 - \frac{50(\cos t - 0.025)}{1.000625} + \frac{50c}{e^{0.025t}}$$

when $y = 150$ and $t = 0$

150 = 20

150 =

15

$$150 = 2000 - \frac{50(1-0)}{1.000625} + \frac{500}{1}$$

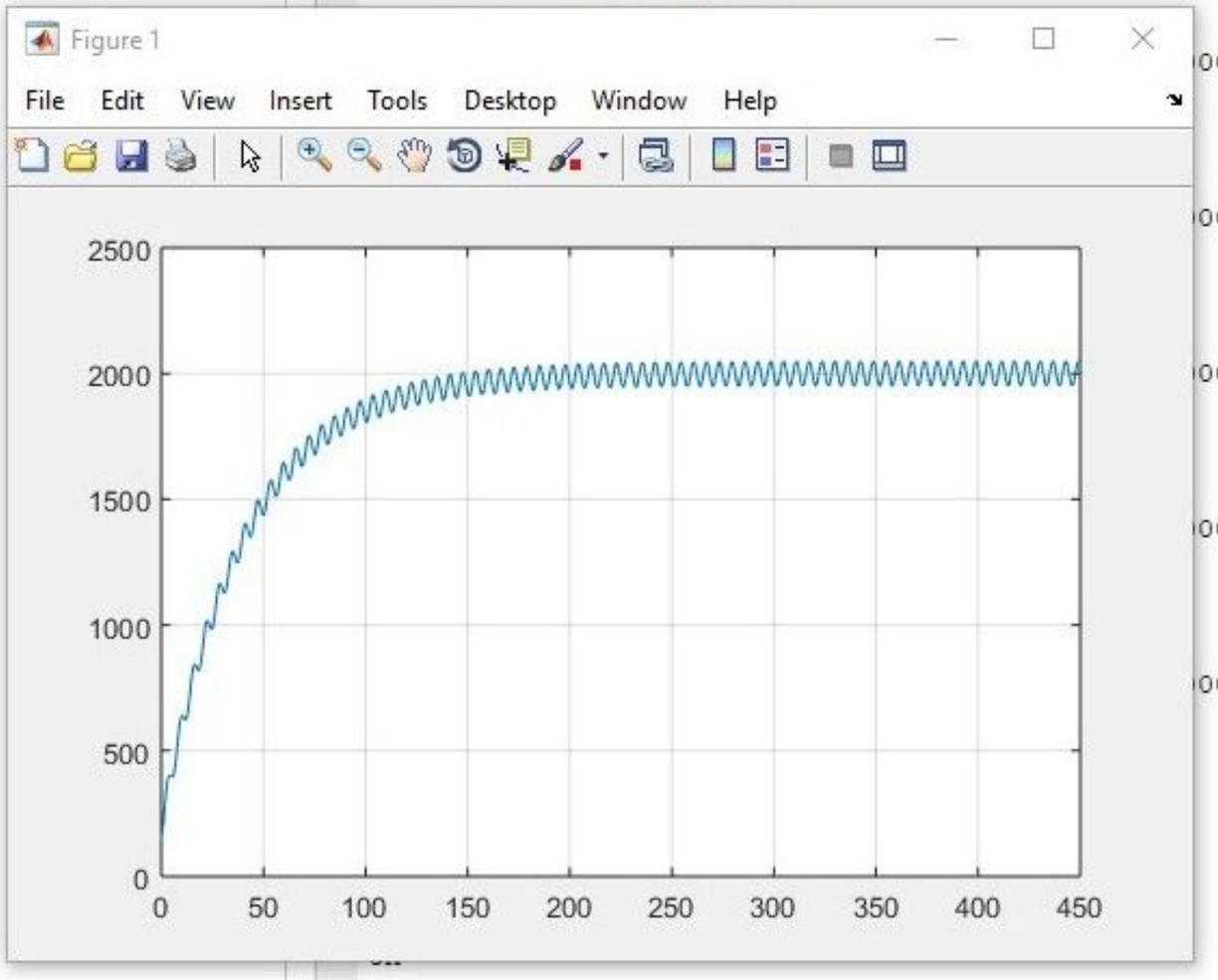
$$150 = 2000 - 49.9(1) + 50c$$

$$150 = 1950.032 + 50$$

$$-1800.032 = 50c$$

$$\therefore c = -36.0064$$

```
commandwindow
clear
clc
close all
syms m t
ans=dsolve('Dm+0.025*m=50+50*sin(t)','m(0)=150')
t=0:0.5:450
tn=subs(ans,t)
plot(t,tn)
grid on
```




```
Columns 876 through 880
[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1
>>
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