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Electrical Electronics Engineering
18/ENG04/027

1. Tank holds 1200gal of water with 150lb of salt dissolved in it.
- Input rate ⇒ 50gal at brine/min [1gal has (1+sint)lb of salt]
- Output rate ⇒ 30gal of brine/min

m ⇒ Amount of salt at anytime t

Using the balance law

$\frac{dm}{dt} = m_{in} - m_{out}$

i. 50gal of water enter per minute and a gallon has (1+sint)lb of salt

$\therefore m_{in} = 50gal/min \times (1+\sin t) = 50(1+\sin t)lb/min$

If the tank contains 1200gal of water and 30gal of water leaves per minute. Theta is $\frac{30}{1200} = \frac{1}{40} = 0.025$

⇒ 2.5% of water in the tank leaves per minute. This also means that 2.5% of salt will also leave the tank per minute

m_{out} ⇒ 2.5% of m

$\frac{dm}{dt} = 50(1+\sin t) - 0.025m$ ——— (i)

b. differentiating using integration factor method

b. $\frac{dm}{dt} + 0.025m = 50(\sin t + 1)$ ——— (ii)

$\because \frac{dy}{dx} + Py = Q \left(\frac{dm}{dt} + Pm = Q\right)$

$P = 0.025, \quad Q = 50(\sin t + 1)$

$e^{\int P dt}$ = Integrating factor

$\int P dt = \int 0.025 dt = 0.025t$

If ⇒ $e^{0.025t}$

From $y \cdot IF = \int Q \cdot IF\, dx$

$m \cdot IF = \int Q \cdot IF\, dt$

$m \cdot e^{0.025t} = \int 50(\sin t + 1) \cdot e^{0.025t}\, dt$

$= 50 \int e^{t/40} (\sin t + 1)\, dt$

Let $u = \frac{t}{40} \rightarrow \frac{du}{dt} = \frac{1}{40}$, $dt = 40\, du$

$= 40 \int e^u (\sin 40u + 1)\, du$

$\int e^u (\sin 40u + 1)\, du$

Let $q = \sin 40u + 1$, $dp = e^u\, du$

$\int dp = \int e^u\, du$

$\frac{dq}{du} = 40 \cos 40u$ $\quad p = e^u$

$dq = 40(\cos 40u)\, du$

From $\int q\, dp = qp - \int p\, dq$

$\int 40e^u (\cos 40u)\, du \Rightarrow 40 \int e^u (\cos 40u)\, du$

$\int e^u \cos 40u\, du \Rightarrow$ Let $q = \cos 40u$ $\quad dp = e^u\, du$

Integrating

$\int e^u \cos 40u\, du = e^u \cos 40u - \int -40e^u (\sin 40u)\, du$ — (iii)

integration by part

$q = -40 \sin 40u$ $\quad dp = e^u$

$dq = -1600(\cos 40u)\, du$ $\quad p = e^u$

$\int e^u \cos 40u\, du = e^u \cos 40u - C - 40e^u \sin 40u - \int -1600 e^u \cos 40u\, du$

$1601 \int e^u \cos 40u\, du = e^u \cos 40 + 40e^u \sin 40u$

$\int e^u \cos 40u\, du = \frac{e^u \cos 40 + 40e^u \sin 40u}{1601}$

$40 \int e^u \cos 40u\, du = 40 (40e^u \sin 40u + e^u \cos 40u)$ — (iv)

From (iii)

$\Rightarrow \int e^u (\sin 40u + 1)\, du = e^u \sin 40u + 1 - \int 40e^u \cos 40u\, du$

$\int e^u (\sin 40u + 1) \Rightarrow e^u \sin 40u + 1 - \frac{40(40e^u \sin 40u + e^u \cos(40u)}{1601}$

$$\int 40e^{u}\sin 40u + 1\,du = 40e^{u}(\sin 40u + 1) - \frac{1600(40e^{u}\sin 40u + e^{u}\cos 40u)}{1601}$$

put $u = t/40$

$$40e^{t/40}(\sin t + 1) - \frac{1600(40e^{t/40} + e^{t/40}\cos(t))}{1601}$$

$$\int_0^t e^{t/40}(\sin t + 1)\,dt$$

$$\int_0^t e^{t/40}(\sin t + 1)\,dt \Rightarrow \frac{2000e^{t/40}(\sin t - 40\cos t + 1601)}{1601} + C \quad \text{— (V)}$$

Recall the original integration factor

$$m.IF = \int Q.IF\,dt$$

$$m \cdot e^{0.025} = \int 150(\sin t + 1)\cdot e^{0.025t}\,dt$$

put eqn (V)

$$m \cdot e^{0.025t} = 2000e^{0.025}\frac{(\sin t - 40\cos t + 1601)}{1601} + C$$

$$m = \frac{2000(\sin t - 40\cos t + 1601)}{1601} + m_0 \cdot e^{-0.025t} \quad \text{— (VI)}$$

at $t = 0$ min and $m = 150$ lb of salt

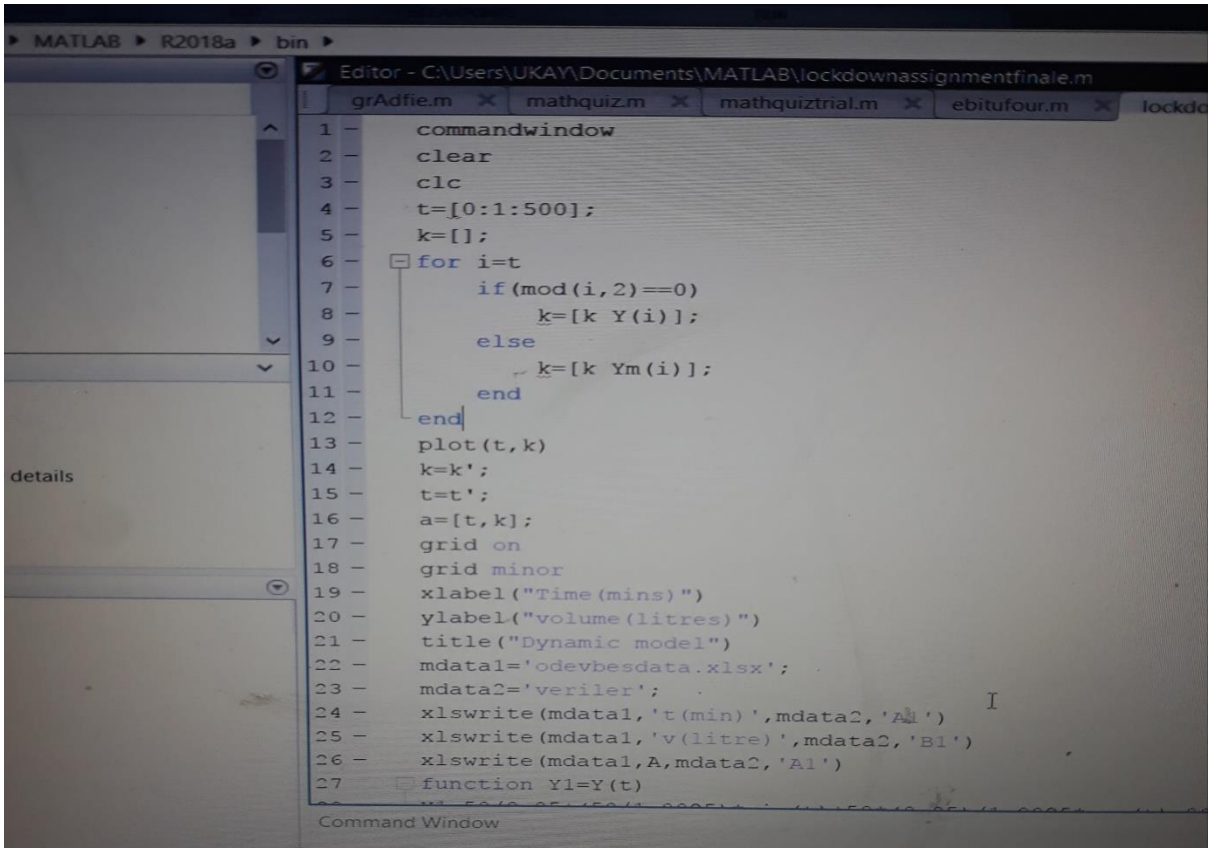
$$150 = \frac{2000(\sin 0 - 40\cos(0) + 1601)}{1601} + m_0 \cdot 1$$

$$150 = 1950.03 + m_0$$

$$m_0 = -1800.03$$

put m_0 in eqn (VI)

$$m = \frac{2000(\sin t - 40\cos t + 1601)}{1601} - \frac{1800.03}{e^{0.025t}}$$



```
        if(mod(i,2)==0)
            k=[k Y(i)];
        else
            k=[k Ym(i)];
        end
    end
    plot(t,k)
    k=k';
    t=t';
    a=[t,k];
    grid on
    grid minor
    xlabel("Time(mins)")
    ylabel("volume(litres)")
    title("Dynamic model")
    mdata1='odevbesdata.xlsx';
    mdata2='veriler';
    xlswrite(mdata1,'t(min)',mdata2,'A1')
    xlswrite(mdata1,'v(litre)',mdata2,'B1')
    xlswrite(mdata1,A,mdata2,'A1')
function Y1=Y(t)
    Y1=50/0.05+(50/1.0025)*sin(t)+50*(0.05)/1.0025*cos(t)-802.49*exp(-0.05*t);
    end
function Ym1=Ym(t)
    Ym1=1000-800*exp(-0.05*t);
    end
```

Command Window