



using balance law

$$\left[\text{Accumulation rate of a Salt within a system} \right] = \left[\text{Input salt rate of the system} \right] - \left[\text{Output rate of Salt from system} \right]$$

$$\text{i.e. } \frac{dm}{dt} = m_{in} - m_{out}$$

$$\frac{dm}{dt} = \left[50(1 + \sin t) \right] - \left[\frac{30}{1200} \times m \right]$$

$$\frac{dm}{dt} = \left[50(1 + \sin t) \right] - \left[\frac{30}{1200} \times m \right] \quad \times m / 40$$

$$\frac{dm}{dt} = 50(1 + \sin t) - \frac{m}{40}$$

~~rearrange the equation~~ \downarrow

$$\frac{dm}{dt} + \frac{m}{40} = 50(1 + \sin t)$$

using integrating factor

$$\int p dt = \int \frac{1}{40} = \frac{t}{40} \therefore I.F. = e^{t/40}$$

since

$$m \cdot I.F. = \int Q \cdot I.F.$$

$$m \cdot e^{t/40} = \int 50(1 + \sin t) \cdot e^{t/40}$$

$$= 50 \int (1 + \sin t) e^{t/40}$$

$$= 50 \left[\int e^{t/40} + \int e^{t/40} \sin t \right]$$

$$= 50 \left(40 e^{t/40} + \int e^{t/40} \sin t \cdot dt \dots \right)$$

Solving
 $\int e^{t/40} \sin t \cdot dt$

Integration by parts: $\int u dv = uv - \int v du$

$$u = \sin t, \quad du = \cos t; \quad dv = e^{t/40} \quad v = 40e^{t/40}$$

$$\therefore \int e^{t/40} \sin t \cdot dt = 40e^{t/40} \times \sin t - \int 40e^{t/40} \times \cos t \cdot dt \quad \text{(ii)}$$

Solving $\int 40e^{t/40} \times \cos t \cdot dt$; Int by parts

$$u = \cos t \quad du = -\sin t; \quad dv = 40e^{t/40} \quad (1600e^{t/40} \text{ ~~cost~~})$$

$$= \int -1600e^{t/40} \sin t$$

$$\therefore \int 40e^{t/40} \cos t \cdot dt = 1600e^{t/40} \cos t - \int 1600e^{t/40} \sin t$$

$$= 1600e^{t/40} \cos t + 1600 \int e^{t/40} \sin t \quad \text{(iii)}$$

Subst (iii) in (ii) due to repetition of $\int e^{t/40} \sin t$

$$\int e^{t/40} \sin t \cdot dt = 40e^{t/40} \sin t - (1600e^{t/40} \cos t + 1600 \int e^{t/40} \sin t$$

$$= 40e^{t/40} \sin t - 1600e^{t/40} \cos t - 1600 \int e^{t/40} \sin t$$

Solving for $\int e^{t/40} \sin t$

$$\int e^{t/40} \sin t \cdot dt + 1600 \int e^{t/40} \sin t \cdot dt$$

$$= 40e^{t/40} \sin t - 1600e^{t/40} \cos t$$

$$1601 \int e^{t/40} \sin t \cdot dt = 40e^{t/40} \sin t - 1600e^{t/40} \cos t$$

$$(iii) \quad \int e^{t/40} \sin t \cdot dt = \frac{40e^{t/40} \sin t - 1600e^{t/40} \cos t}{1601}$$

putly $e^{t/40}$ in \times

$$m \cdot e^{t/40} = 50(40)^{t/40} = 50(40e^{-t/40} + 40$$

$$+ 40e^{t/40} \sin t - (8000e^{-t/40}) \text{ cost}$$

1601

~~$m \cdot e^{t/40} = 2000e^{t/40} + 2000e^{t/40} \sin t - 8000e^{-t/40}$~~

$$m \cdot e^{t/40} = 2000e^{t/40} + 2000e^{t/40} \sin t - 8000e^{-t/40} \text{ cost}$$

1601

+ C

dividing both sides by $e^{t/40}$

$$m = 2000 + 2000 \sin t - 80000 \text{ cost} + Ce^{-t/40}$$

1601

+ $Ce^{-t/40}$

$C = m_0$ when $t = 0$ $m = 150$

$$150 = 2000 + \frac{-80000}{1601} + m_0$$

$$150 = 1950 + m_0$$

$$m_0 = 150 - 1950$$

$$m_0 = -1800$$

$$\therefore \text{Eqn for } m = 2000 + 2000 \sin t - 80000 \text{ cost} -$$

1601

$$- 1800e^{-t/40}$$

at $t = 0$; $m = 150$; $A = 1800$

```

1  commandwindow
2  clear
3  clc
4  close all
5  syms m t
6  ans=dsolve('Dm+0.025*m=50+50*sin(t)', 'm(0)=150')
7  t=0:0.5:450
8  tn=subs(ans,t)
9  plot(t,tn)

```

Command Window

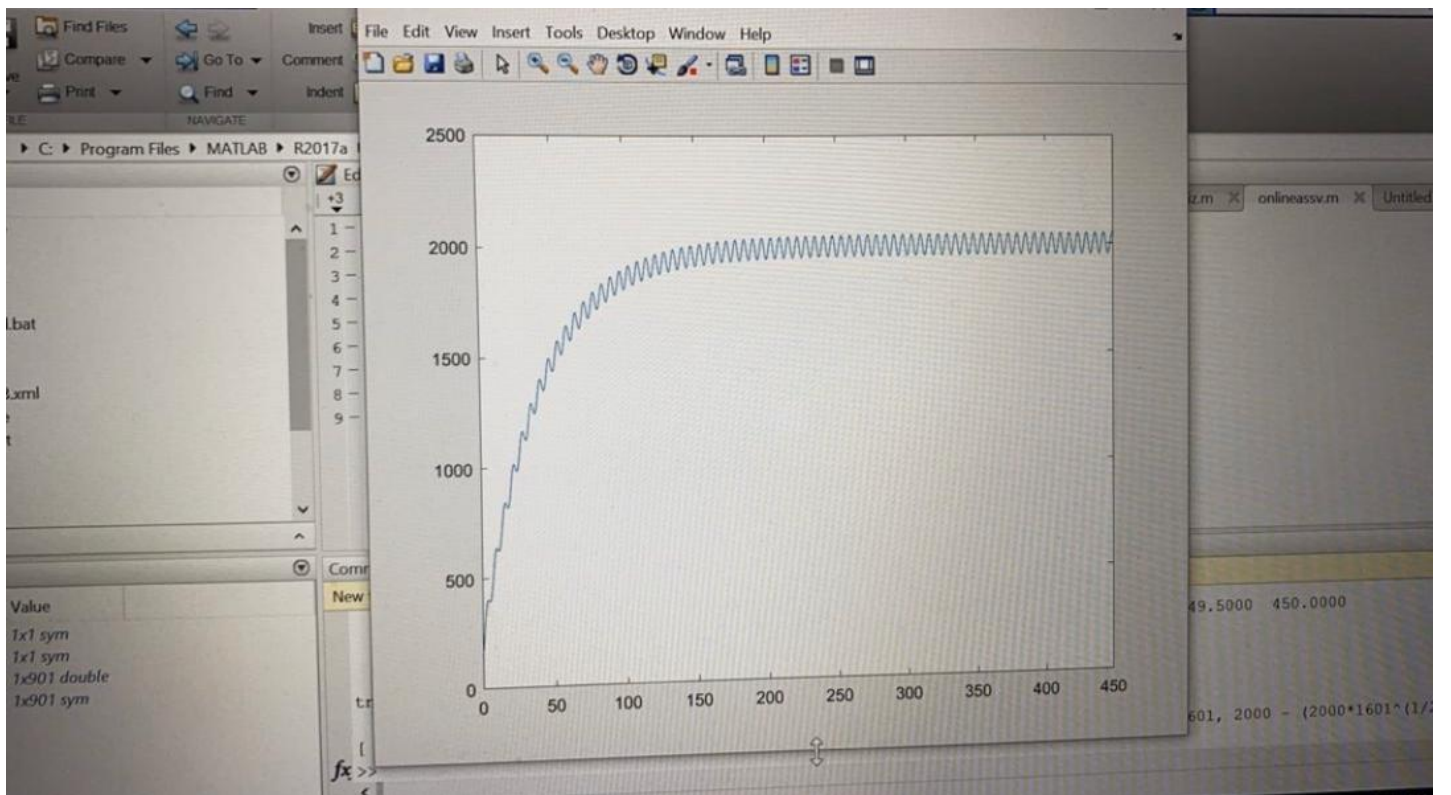
New to MATLAB? See resources for [Getting Started](#).

445.5000 446.0000 446.5000 447.0000 447.5000 448.0000 448.5000 449.0000 449.5000 450.0000

tn =

[150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, ...

fx>>



```

1 - commandwindow
2 - clc
3 - clear all
4 - close all
5 - t=0:1:499
6 - t1=1:2:499
7 - t2=0:2:500
8 - y= (50/0.05)+((50/1.0025)*sin(t1))+(((50*0.05)/1.0025)*cos(t1))-(802.4*exp(-0.05*t1))
9 - ym= 1000-(800*exp(-0.05*t2))
10 - T1=t1'
11 - T2=t2'
12 - Y=y'
13 - YM=ym'
14 - values=[]
15 - A=[Y']
16 - B=[YM']
17 - i=1

```

```

15 - A=[Y']
16 - B=[YM']
17 - i=1
18 - j=1
19 - while (i<=250 && j<=250)
20 -     Bn=B(j)
21 -     values=[values;Bn]
22 -     An=A(i)
23 -     values=[values;An]
24 -     i=i+1;
25 -     j=j+1;
26 - end
27 - plot(t,values,'black')
28 - grid on
29 - grid minor
30 - xlabel('t (min)')

```

