WAKAMA FESTA NENGI
18/SC114/025
CIVIL ENGINEERING
ENG 282
ENGINEERING MATHS II

1. % output rate of water = % output rate of salt

$\frac{80 gal}{1200 gal/min} = \frac{1}{40} \times 100\% = 2.5\%$ per min

$2.5\% = 0.025m$ lb/min

$\therefore$ rate at which it is accumulated $\left(\frac{dm}{dt}\right) = 50(1+\sin t)\frac{lb}{min} - 0.025m\ \frac{lb}{min}$

$\Rightarrow \frac{dm}{dt} = 50(1+\sin t) - 0.025m$

$\Rightarrow \frac{dm}{dt} = -0.025m + 50(1+\sin t)$

$\Rightarrow \frac{dm}{dt} + 0.025m = 50(1+\sin t)$

$$\boxed{\frac{dy}{dx} + Py = Q}$$ whose $P = 0.025$, $Q = 50(1+\sin t)$

Using Integrating factor method.

$IF = e^{\int P dx} = e^{\int 0.025 dt} = e^{0.025t}$

$Y \cdot IF = \int Q \cdot IF\, dx$

$= m \cdot IF = \int Q \cdot IF\, dt$

$$\Rightarrow m \cdot e^{0.025t} = \int Q \cdot e^{0.025t}\, dt$$

$$\Rightarrow m \cdot e^{0.025t} = \int 50(1+\sin t)\, e^{0.025t}\, dt$$

$$\Rightarrow m \cdot e^{0.025t} = 50 \int (1+\sin t) e^{0.025t}\, dt \quad \text{---} \ (1)$$

Integration by parts

$v = (1+\sin t)$; $dv = \cos t$, $du = e^{0.025t}$; $u = e^{0.025t}/0.025$

$$\int v\, du = uv - \int u\, dv$$

$$\therefore \left(\frac{e^{0.025t}}{0.025}(1+\sin t)\right) - \frac{1}{0.025}\int e^{0.025t}\cos t$$

Integrating $e^{0.025t}\cos t$

$$\int e^{0.025t}\cos t = \frac{1}{1601} \times \frac{e^{0.025t}}{0.025}\left(\cos t + \frac{\sin t}{0.025}\right)$$

$$\int e^{0.025t}\cos t = \frac{e^{0.025t}}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)$$

$$\therefore \int (1+\sin t)\, e^{0.025t}$$

$$\Rightarrow \left(\frac{e^{0.025t}}{0.025}(1+\sin t)\right) - \frac{1}{0.025}\left(\frac{e^{0.025t}}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)\right)$$

$$\therefore 50\int (1+\sin t)\, e^{0.025t}$$

$$\Rightarrow \frac{50}{0.025}\left(\left(\frac{e^{0.025t}}{e^{0.025t}}(1+\sin t)\right) - \left(\frac{e^{0.025t}}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)\right)\right)$$

$$\Rightarrow 2000\, e^{0.025}\left((1+\sin t) - \frac{1}{40.025}\left(\cos t + \frac{\sin t}{0.025}\right)\right)$$

$$\cos t + \frac{\sin t}{0.025} = \frac{0.025\cos t + \sin t}{0.025}$$

$$\Rightarrow 2000\, e^{0.025t}\left((1+\sin t) - \frac{1}{40.025}\left(\frac{0.025\cos t + \sin t}{0.025}\right)\right)$$

$$\Rightarrow 2000\, e^{0.025t}\left((1+\sin t) - \frac{1}{1.000625}(0.025\cos t + \sin t)\right)$$

$$\therefore m \cdot e^{0.025t} = 2000\, e^{0.025t}\left((1+\sin t) - \frac{1}{1.000625}(0.025\cos t + \sin)\right)$$

$$+ \frac{c}{e^{0.025t}}$$

$t = 0, \quad m = 150 lb$

$$\Rightarrow 150 = 2000\left(\left(1 + \sin(0) - \frac{1}{1.000625}\left(0.025\cos(0) + \sin(0)\right)\right) + \frac{C}{e^{0.025(0)}}\right.$$

$$\Rightarrow 150 = 2000\left(1 - \frac{1}{1.000625}\left(0.025(1) + 0\right)\right) + \frac{C}{e^{0}}$$

$$\Rightarrow 150 = 2000\left(1 - 0.025/1.000625\right) + C$$

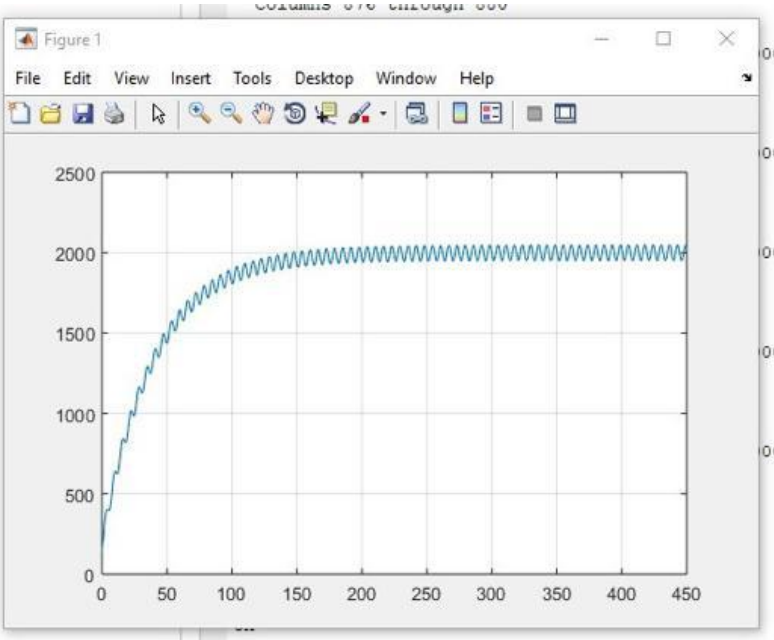
$$\Rightarrow C = 150 - 2000\left(1 - \frac{0.025}{1.000625}\right)$$

$$\Rightarrow C = 150 - 1950.0312304$$

$$\Rightarrow -1800.031$$

$$\therefore m = \left((1 + \sin t) - \left(\frac{0.025\cos t + \sin t}{1.000625}\right)2000\right) - \frac{1800.03}{e^{0.025t}}$$

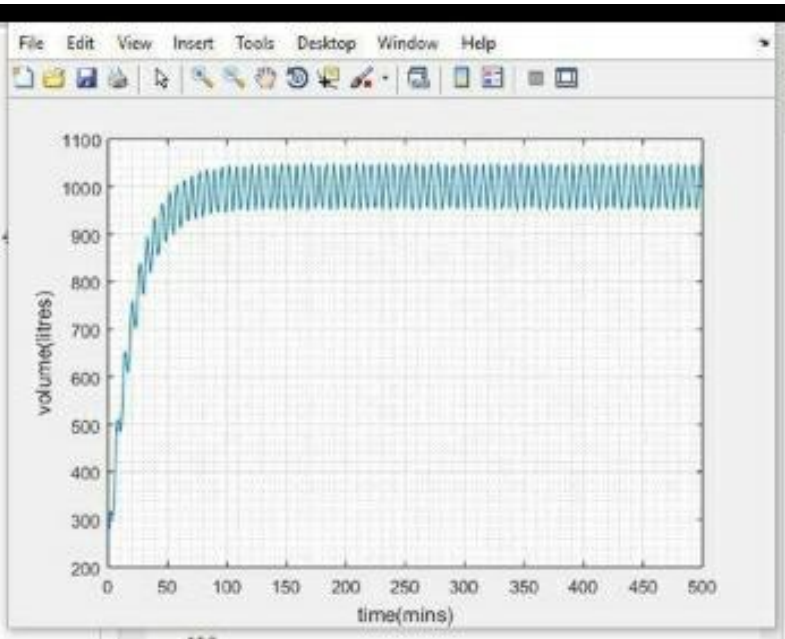
```
commandwindow
clear
clc
close all
syms m t
ans=dsolve('Dm+0.025*m=50+50*sin(t)','m(0)=150')
t=0:0.5:450
tn=subs(ans,t)
plot(t,tn)
grid on
```




```
[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1
>>
```

```
commandwindow
clear
clc
close all
syms t
values=[]
t=1:1:500
mean=1000-((exp(-0.05*t))*800)
y=1000+(50/1.0025)*sin(t)+(2.5/1.0025)*cos(t)-((exp(-0.05*t))*802.4

if rem(t,2) ==0
    values=[values,mean]
else
    values=[values,y]
end
excelvalues=transpose(values)
mins=transpose(t)
plot(t,values)
grid on
grid minor
xlabel ('time(mins)')
ylabel('volume(litres)')
xlswrite('odevbesdata.xlsx',{'t(min)'},'veriler','A1')
xlswrite('odevbesdata.xlsx',mins,'veriler','A2')
xlswrite('odevbesdata.xlsx',{'V(Litre)'},'veriler','B1')
xlswrite('odevbesdata.xlsx',excelvalues,'veriler','B2')
```




```
498
499
500
>>
```