Name: OKWUOKWU BRYAN COURSE: MAT 104

MATRIC NO: 19/ENG05/1049 DEPARTMENT: Mechatronics

① $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{A}{x-1} + \int \frac{B}{x-2} + \int \frac{C}{x-3}$

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x-1$

$3(1)-1 = A(1-2)(1-3) \Rightarrow 2 = A(-1)(-2) = 2A$

$\therefore \frac{2A}{2} = \frac{2}{2} \quad \therefore A = 1$

Put $x=2$

$3(2)-1 = B(2-1)(2-3)$

$5 = B(1)(-1) \quad 5 = -B \quad \therefore B = -5$

Put $x=3$

$3(3)-1 = C(3-1)(3-2)$

$9-1 = C(2)(1) \Rightarrow \frac{8}{2} = \frac{2C}{2} \quad \therefore C = 4$

$\int \frac{3x-1}{(x-1)(x-2)(x-3)} = \int \frac{1}{x-1} - \int \frac{5}{x-2} + \int \frac{4}{x-3}$

$= \ln|x-1| - 5\ln|x-2| + 4\ln|x-3| + C$

(2) $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{A}{x+2} + \int \frac{Bx+C}{x^2+1}$

$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)(x+2) + (Bx+C)(x+2)}{(x+2)(x^2+1)}$

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

Putting $x=-2$

$(-2)^2 + (-2) + 1 = A((-2)^2+1) + 0$

$4-2+1 = 5A \quad \therefore A = \frac{3}{5}$

Putting $x=0$

$0+0+1 = A((0)+1) + (0+C)(0+2)$

$1 = A + 2C = \frac{3}{5} + 2C$

$2C = 1 - \frac{3}{5} = \frac{2}{5} \qquad \frac{2C}{2} = \frac{1}{5} \div 2$

$\therefore C = 1/5$

Putting $x=1$; $1+1+1 = 2A + (B+C)3$

$3 = 2A + 3(B+C) \qquad 3 = 2(\frac{3}{5}) + 3(B + \frac{1}{5})$

$3 - 6/5 = 3(B + 1/5) \qquad 9/5 - \frac{3}{5} = 3B$

$\therefore B = 2/5$

$$\therefore \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5(x+2)} + \frac{1(2x+1)}{5(x^2+1)}$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\int \frac{1}{x+2}\,dx + \frac{1}{5}\int \frac{2}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}\,dx$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\ln|x+2| + \frac{1}{5}\ln|x^2+1| + \frac{1}{5}\tan^{-1}(x) + C$$

(3) $$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{A}{x-3}\,dx + \int \frac{B}{(x-2)}\,dx + \int \frac{C}{(x-2)^2}\,dx$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-2)(x-3) + C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = A(x-2)^2 + B(x-2)(x-3) + C(x-3) \Rightarrow (1)$$

$$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + Cx - 3C$$

$$x^2+1 = Ax^2 + Bx^2 - 4Ax - 5Bx + Cx + 4A + 6B - 3C$$

Using eqn. (1)

Put $x = 2$

$$(2)^2+1 = 0 + 0 + C[2-3] \Rightarrow \frac{5}{-1} = \frac{-C}{-1}$$

$$\therefore C = -5$$

Put $x = 3$

$(3)^2 + 1 = A(3-2)^2 + 0 + 0$

$10 = A \quad \therefore A = 10$

Using eqn. 2; $A + B = 1 \Rightarrow B = 1 - A$

$\therefore B = 1 - 10 = -9$

$$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{10}{(x-3)}\,dx + \int \frac{(-9)}{(x-2)}\,dx + \int \frac{(-5)}{(x-2)^2}$$

$$= 10 \ln|x-3| + (-9)\ln|x-2| + \frac{5}{3(x-2)^3}$$

(4) $$\int \frac{x^3+x^2+x+1}{(x-1)}\,dx$$

$$\begin{array}{r|l} & x^2+2x+3 \\ \hline x-1 & x^3+x^2+x+1 \\ & -(x^3-x^2) \\ \hline & 2x^2+x+1 \\ & -(2x^2-2x) \\ \hline & 3x+1 \\ & -(3x-1) \\ \hline & 2 \end{array}$$

$$\therefore \int \frac{x^3+x^2+x+1}{(x-1)}\,dx = \int x^2+2x+3\,dx + \int \frac{2}{x-1}$$

$$= \frac{x^3}{3} + x^2 + 3x + 2\ln|x-1| + C$$