18/ENG03/038

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1. According to Balance Law, the rate of acceleration of salt within a system is equal to the Input rate of salt into the system subtracting the output rate.

$\therefore I \cdot R \cdot S - O \cdot R \cdot S$

Let salt present in the tank at any time be 'y'.

Time Rate of change of $y = \frac{dy}{dt} = y_{in} - y_{out}$

50 gal/min & each gallon has $(1 + \sin t)$ lb of salt,

at $t = 1$, $(1 + \sin t) = (1 + \sin(1)) = 1.0216$

$50\ gal/min \times 1.0216\ lb/gal$

$= \frac{50\ gal}{min} \times \frac{1.0216\ lb}{gal} = \frac{51.6\ lb}{min} = 51.6\ lb/min$

(Amount of salt that enters the tank)

The tank has 1200 gal of salt solution and 3 gal of the solution leaves the tank every minute: $\frac{3}{1200}$ (gal) = 0.025 - 2.5% of the content of the tank. Therefore 2.5% of the salt in the tank will also leave the tank every minute.

y_{out} = 2.5% of y

a) $\frac{dy}{dt}$ lb/min = 5 lb/min - 2.5% of y lb/min

b) $\frac{dy}{dt} = 5 - 0.025y$; $\frac{dy}{dt} = -0.025y + 5$

$$\frac{dy}{dt} = -0.025\left(\frac{-0.025y}{-0.025} + \frac{5}{-0.025}\right)$$

$$\frac{dy}{dt} = -0.025(y - 200)$$

$$\frac{dy}{(y-200)} = -0.025dt$$

$$\int \frac{dy}{(y-200)} = \int -0.025dt$$

$$\int \frac{dy}{(y-2040)} = -0.025 \int dt \; ; \; \ln(y-2040)$$

$$= -0.025t + C$$

$$y - 2040 = e^{-0.025tC} \; ; \; y - 2040 = e^{-0.025t} e^{C}$$

$$= y - 2040 = e^{-0.025t} \cdot y_0$$

$$= y - 2040 = y_0 e^{-0.025t}$$

$$y - y_0 e^{-0.025t} + 2040 \; ; \text{ initially}$$

when $t = 1$, $y = 1506$

$$= 150 = y_0 e^{-0.025t} + 2040,$$

$$= 150 - 2040 = y_0 \times 1$$

$$y_0 = -1890$$

$$y = -1890 e^{-0.025t} + 2040$$

$$y = 2040 - 1890 e^{-0.025t}$$