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Council for the Regulation of Engineering in Nigeria
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COREN

18/ENG04/7029
Elect-Elect
ENG 282

Engineering Mathematics II

1) Accumulation rate of salt = Input rate of salt into the system - Output rate of salt from the system

Let m be the amount of salt present in the tank at time t

Gallon of water in tank = 1200 gal

Amount of salt initially dissolved = 150 lb

Inflow rate of brine = 50 gal/min

Amount of dissolved salt in a gallon = $\left(1+\frac{1}{2}\sin t\right)$ lb/gal

Outflow rate of brine = 30 gal/min

$\frac{dm}{dt} = m_{in} - m_{out}$

m_{in} = Inflow rate of brine × Amount of salt in a gallon

$m_{in} = 50\ \frac{gal}{min} \times \left(1+\frac{1}{2}\sin t\right)\ \frac{lb}{gal}$

$m_{in} = 50\left(1+\frac{1}{2}\sin t\right)$ lb/min

$m_{out} = \frac{\text{Outflow rate of brine}}{\text{Total gallon of water}}$

$$M_{out} = \frac{30}{1200} = 0.025 = 2.5\% \text{ of } M \text{ in tank}$$

$$\frac{dm}{dt} = 50(1+\sin t) - 0.025m$$

$$\frac{dm}{dt} + 0.025m = 50(1+\sin t)$$

Using integrating factor (I.F)

$$\frac{dy}{dx} + Py = Q$$

$$\frac{dm}{dt} + Pm = Q$$

$$P = 0.025, \quad Q = 50(1+\sin t)$$

$$\int P\,dt = \int 0.025\,dt = 0.025t$$

$$IF = e^{\int P\,dt} = e^{0.025t}$$

$$m \cdot IF = \int Q \cdot IF\,dt$$

$$m \cdot e^{0.025t} = \int 50(1+\sin t) \cdot e^{0.025t}\,dt$$

$$m \cdot e^{0.025t} = \int 50 + 50\sin t \cdot e^{0.025t}\,dt$$

$$m \cdot e^{0.025t} = \int 50e^{0.025t} + 50\sin t\, e^{0.025t}\,dt$$

$$m \cdot e^{0.025t} = \int 50e^{0.025t} + \int 50\sin t\, e^{0.025t}$$

$$m \cdot e^{0.025t} = \frac{50e^{0.025t}}{0.025} + \int 50\sin t\, e^{0.025t}$$

$\int 50 \sin t\, e^{0.025t} = 50 \int \sin t\, e^{0.025t}$

Using integration by parts

Let $u = e^{0.025t}$, $dv = \sin t$, $du = 0.025 e^{0.025t}$, $v = -\cos t$

$\int u\,dv = uv - \int v\,du$

$\int \sin t\, e^{0.025t} = e^{0.025t}(-\cos t) - \int -\cos t\,(0.025 e^{0.025t})$

$= -e^{0.025t} \cos t + 0.025 \int e^{0.025t} \cos t$

$\int e^{0.025t} \cos t =$

$u = e^{0.025t}$, $dv = \cos t$, $du = 0.025 e^{0.025t}$, $v = \sin t$

$\int e^{0.025t} \cos t = e^{0.025t}(\sin t) - \int \sin t\,(0.025 e^{0.025t})$

$= e^{0.025t} \sin t - 0.025 \int e^{0.025t} \sin t$

$\int \sin t\, e^{0.025t} = -e^{0.025t} + 0.025 e^{0.025t} \sin t - 0.000625 \int \sin t\, e^{0.025t}$

Using I for the integral of $\int e^{0.025t} \sin t$

$I = -e^{0.025t} \cos t + 0.025 e^{0.025t} - 0.000625 I$

$1.000625 I = e^{0.025t}(0.025 \sin t - \cos t) + C_1$

$I = \frac{e^{0.025t}(0.025 \sin t - \cos t)}{1.000625} + C$

$50 \int \sin t\, e^{0.025t} = 50 \left[\frac{e^{0.025t}(0.025 \sin t - \cos t)}{1.000625} + C \right]$

$$m \cdot e^{0.025t} = 2000e^{0.025t} + 50\left(\frac{e^{0.025t}}{1.000625}(0.025\sin t - \cos t) + C\right)$$

$$m = \frac{2000e^{0.025t}}{e^{0.025t}} + \frac{50}{e^{0.025t}}\left[\frac{e^{0.025t}}{1.000625}(0.025\sin t - \cos t) + \frac{C}{e^{0.025t}}\right.$$

$$m = 2000 + 49.9687(0.025\sin t - \cos t) + \frac{C}{e^{0.025t}}$$

When $t = 0$, $y = 150$

$$150 = 2000 + 49.9687(0.025\sin 0 - \cos 0) + \frac{C}{e^{0.025(0)}}$$

$$150 = 2000 + 49.9687(0 - 1) + C$$

$$150 = 2000 - 49.9687 + C$$

$$C = 150 - 2000 + 49.9687$$

$$C = -1800.0315$$

$$m = 2000 + 49.9687(0.025\sin t - \cos t) + \frac{1800.0315}{e^{0.025t}}$$

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matlab.mat (MAT-file)

Workspace

Name	Value
A	1x1 sym
An	1x901 sym
cond	1x1 sym
equ	1x1 symfun
m	1x1 symfun
t	1x901 double

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```
1 -    commandwindow
2 -    clc
3 -    clear
4 -    close all
5 -    syms m(t) t equ
6 -    syms A(t)
7 -    equ = diff(m,t) == 50*(1 + sin(t)) - 0.025*m;
8 -    cond = m(0)==150;
9 -    A = dsolve(equ,cond)
10 -   t = [0:0.5:450]
11 -   An=subs(A,t)
12 -   plot(t,An,'black')
13 -   ylabel('Amount of salt in the tank(lb)')
14 -   xlabel('Time(mins)')
15 -   grid on
16 -   grid minor
```

Command Window

```
   450.0000


An =

[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, 2000 - (2000*1601^(1/2)*cos(atan(1
fx >>
```

Figure 1

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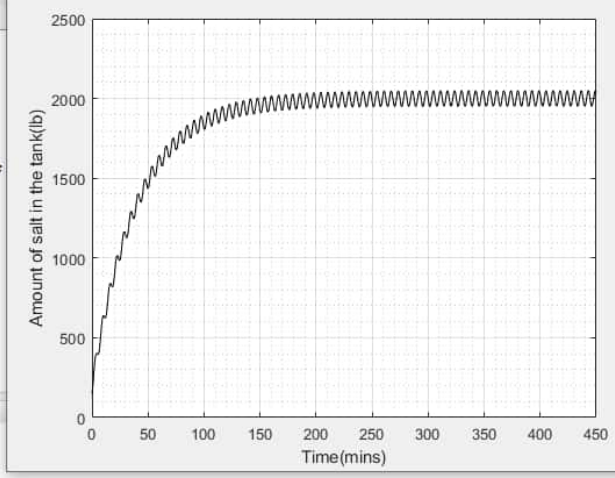

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```
1 -    commandwindow
2 -    clc
3 -    clear
4 -    close all
5 -    syms m(t) t equ
6 -    syms A(t)
7 -    equ = diff(m,t) == 50*(1 + sin(t)) - 0.025*m;
8 -    cond = m(0)==150;
9 -    A = dsolve(equ,cond)
10 -   t = [0:(0.5/60):7.5]
11 -   An=subs(A,t)
12 -   plot(t,An,'black')
13 -   ylabel('Amount of salt in the tank(lb)')
14 -   xlabel('Time(hrs)')
15 -   grid on
16 -   grid minor
```

Figure 1

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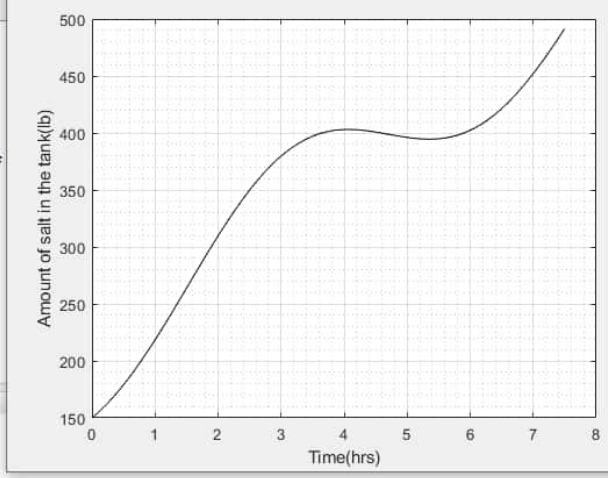

Command Window

```
    7.5000


An =

[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/120))/1601 - (2881850*exp(-1/4800))/1601, 2000 - (2000*1601^(1/2)*cos(at
>>
```

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