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Computer Engineering

1a) from $\frac{dy}{dx} = y_{in} - y_{out}$
 $\frac{dy}{dt} = 50(1 + \sin t) - 2.5\% \text{ of } y$

$$y_{out} = \frac{30}{1200}$$

$$y_{out} = 2.5\% y = \frac{1}{40} y$$

$$\frac{dy}{dt} = 50(1 + \sin t) - 0.025y$$

$$\frac{dy}{dt} + 0.025y = 50(1 + \sin t)$$

$$\Rightarrow 1 + 0.025y = 50(1 + \sin t) e^{\int 0.025 dt}$$

1b) $\frac{dy}{dt} = 50(1 + \sin t) - 0.025y$

using

$$\frac{dy}{dt} + Py = Q$$

$$P = 0.025, Q = 50(1 + \sin t)$$

$$P \cdot dt = 0.025 dt$$

$$I.F = e^{\int P dt}$$

$$I.F = e^{0.0025 dt}$$

$$y \cdot e^{0.025t} = \int 50(1 + \sin t) e^{0.025t} dt$$

$$y \cdot e^{0.025t} = 50 \int (1 + \sin t) e^{0.025t} dt$$

$$y \cdot e^{0.025t} = 50 \int e^{0.025t} dt - \int e^{0.025t} \sin t dt$$

$$y \cdot e^{0.025t} = \frac{50 e^{0.025t}}{0.025} + \int e^{0.025t} \sin t dt$$

Integration by part

$$u = e^{0.025t} \quad dv = \sin t$$

$$du = 0.025 e^{0.025t} \quad v = -\cos t$$

$$\int e^{0.025t} \sin t = e^{0.025t} (-\cos t) - \int 0.025 e^{0.025t} (-\cos t) dt$$

$$\int e^{0.025t} \sin t = -e^{0.025t} \cos t + 0.025 \int e^{0.025t} \cos t dt$$

$$\int u dv = uv - \int v du$$

$$u = e^{0.025t} \quad du = 0.025 e^{0.025t} dt$$

$$u = 6.025 \int \frac{1}{u} du = 6.025 \ln|u| + C$$

$$= -e^{0.025t} \int (0.025 e^{0.025t} \sin t - 0.025 e^{0.025t}) dt$$

$$\text{Let } q = \int e^{0.025t} \sin t$$

$$q = -e^{0.025t} \cos t + 0.025 \int e^{0.025t} \sin t dt - 0.025 q$$

$$q = -e^{0.025t} \cos t + 0.025 \int e^{0.025t} \sin t dt - 0.025 q$$

$$q + 6.25^{-1} q = -e^{0.025t} \cos t + 0.025 e^{0.025t}$$

$$1.000625 q = -e^{0.025t} \cos t + 0.025 e^{0.025t}$$

$$q = \frac{-e^{0.025t} \cos t + 0.025 e^{0.025t}}{1.000625}$$

$$y = 2000 - 50 \int \frac{1}{1.000625} (0.025 e^{0.025t} \sin t - 0.025 e^{0.025t}) dt + 50 \int e^{0.025t}$$

$$\text{When } y = 150, t = 0$$

$$150 = 2000 - \frac{50}{1.000625} (1 - 0.9) + 50 \int$$

$$150 = 2000 - 44.400(1) + 50 \int$$

$$150 = 1750.022 + 50 \int$$

$$\frac{50 \int}{50} = \frac{-1800.022}{50}$$

$$C = -36.00064$$

$$q = \frac{-e^{0.025t} \cos t + 0.025 e^{0.025t}}{1.000625}$$

$$\int e^{0.025t} \sin t = \frac{-e^{0.025t} \cos t + 0.025 e^{0.025t}}{1.000625}$$

$$y \cdot e^{0.025t} = 50 \int \frac{e^{0.025t}}{0.025} - \frac{e^{0.025t}}{1.000625} [(0.025 - 0.025)t + C]$$

$$y \cdot e^{0.025t} = 2000e^{0.025t} - \frac{50 \cdot e^{0.025t}}{1.000625} [(0.025 - 0.025)t + C]$$

$$y = 2000 - \frac{50}{1.000625} [(0.025 - 0.025)t + \frac{50C}{e^{0.025t}}]$$