NAME:

SUNDAY WINNER CHIGOZIRIM

COURSE:

ENGINEERING MATHEMATICS II (ENG282)

MATRIC:

18/ENG05/057

DEPT:

MECHATRONICS ENGINEERING

N.B: MATLAB mobile was used in the solution of the Assignment V given on the 29th of April 2020

QUESTION 1:

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ASSIGNMENT 4 SOLUTION.

(1a) Water amount = 1200 gal

Salt amount = 150 lb

Input rate = 50 gal/min ; (1 + sin t) lb in each gallon

Output rate = 30 gal/min;

$$\frac{dy}{dx} = y_{in} - y_{out} \; ; \; y_{in} = \frac{50\,gal}{1\,min} \times \frac{(1 + \sin t)}{1\,gal} = (50 + 50\sin(t))\,lb/min$$

$$y_{out} = \frac{30}{1200} = 0.025 = 2.5\% \; \text{of } y$$

$$\frac{dy}{dx} = (50 + 50\sin(t)) - 0.025y \implies \text{Ordinary differential equation}$$

(b) $\dfrac{dy}{dx} + 0.025y = (50 + 50\sin(t))$

from $\dfrac{dy}{dx} + Py = Q \; ; \; \therefore P = 0.025$

$$\int P\,dt = 0.025t$$

$$IF = e^{\int P\,dt} = e^{0.025t}$$

$$\therefore y \cdot IF = \int Q \cdot IF$$

$$y \cdot e^{0.025t} = \int (50 + 50\sin(t)) \, e^{0.025t}$$

INTEGRATING THE RHS

$$\int 50\,e^{0.025t} + 50\,e^{0.025t}\,\sin(t)\,dt$$

$$\implies 2000\,e^{0.025t} + \int 50\,e^{0.025t}\,\sin(t)\,dt$$

INTEGRATING $\int 50\,e^{0.025t}\,\sin(t)$ where $\int 50\,e^{0.025t}\,\sin(t) = I \dots (1)$

let $u = \sin t$ $dv = e^{0.025t}\,dt$

$du = \cos t\,dt$ $v = \dfrac{e^{0.025t}}{0.025}$

$$= 50\left(\sin(t)\frac{e^{0.025t}}{0.025} - \int \frac{e^{0.025t}}{0.025}(\cos(t))\,dt\right) \text{ from } \int u\,dv = uv - \int v\,du$$

$$= 2000\sin(t)\,e^{0.025t} - \int 2000\,e^{0.025t}\cos(t) \dots (2)$$

INTEGRATING $\int 2000 \, e^{0.025t} \cos t$

$\Rightarrow 2000 \int e^{0.025t} \cos t$

let $u = \cos t$ $\qquad dv = e^{0.025t} \, dt$

$du = -\sin t$ $\qquad v = 40 \, e^{0.025 \frac{t}{8}}$

from $\int u \, dv = uv - \int v \, du$

$= 2000 \left(40 e^{0.025t} \cos(t) - \int 40 e^{0.025t} - \sin t \, dt \right)$

$= 80,000 \, e^{0.025t} \cos(t) + 80,000 \int \sin t \, dt \, e^{0.025t}$

$= 80,000 \, e^{0.025t} \cos(t) + 1600 \int 50 \sin t \, e^{0.025t} \, dt$

Recall that $\int 50 \sin t \, e^{0.025t} = I$

$\therefore \quad = 80,000 \, e^{0.025t} \cos(t) + 1600 \, I$

from equation (2)

$\int 50 \sin t \, e^{0.025t} = 2000 \sin(t) e^{0.025t} - \left(80,000 \, e^{0.025t} \cos(t) + 1600 \, I \right)$

$I = 20,000 \sin(t) e^{0.025t} - 80,000 \, e^{0.025t} \cos(t) - 1600 \, I$

$1601 \, I = 20,000 \sin(t) e^{0.025t} - 80,000 \, e^{0.025t} \cos(t)$

$1601 \, I = \exp(t/40) \left(2000 \sin(t) - 80,000 \cos(t) \right)$

$1601 \, I = \exp(t/40) \left(80,000 \left[\frac{\sin(t)}{40} - \cos(t) \right] \right)$

$I = \exp(t/40) \left(80,000 \left[\frac{\sin(t)}{40} - \cos(t) \right] \right) \times \frac{1}{1601}$

from $y \cdot IF = \int \varphi \cdot IF$

$y \cdot \exp(t/40) = 2000 \exp(t/40) + \dfrac{\left(\left[\frac{\sin(t)}{40} - \cos(t) \right] 80,000 \right) \exp(t/40)}{1601}$

$y = 2000 + \dfrac{\left(\left[\frac{\sin(t)}{40} - \cos(t) \right] 80,000 \right) \times \frac{1}{1601}}{}$

$m = 2000 + \frac{1}{1601} \left(80,000 \left(\frac{\sin(t)}{40} - \cos(t) \right) \right)$

QUESTION 1C:

```
 1 commandwindow
 2 clearvars
 3 clc
 4 close all
 5
 6 syms m(t) t
 7 eqn = diff(m,t) == (50+50*sin(t))-0.025*
 8 cond = m(0)==150;
 9 mSol(t) = dsolve(eqn,m(0)==150)
10 t = 0:0.5:450;
11 plot(t,mSol(t))
12 xlabel('Time(hr)')
13 ylabel('Amount of substance(lb)')
14 grid on
15 grid minor
```

←   **Figures**   SHOW DATA CURSOR   ⋮

Figure 1: number2
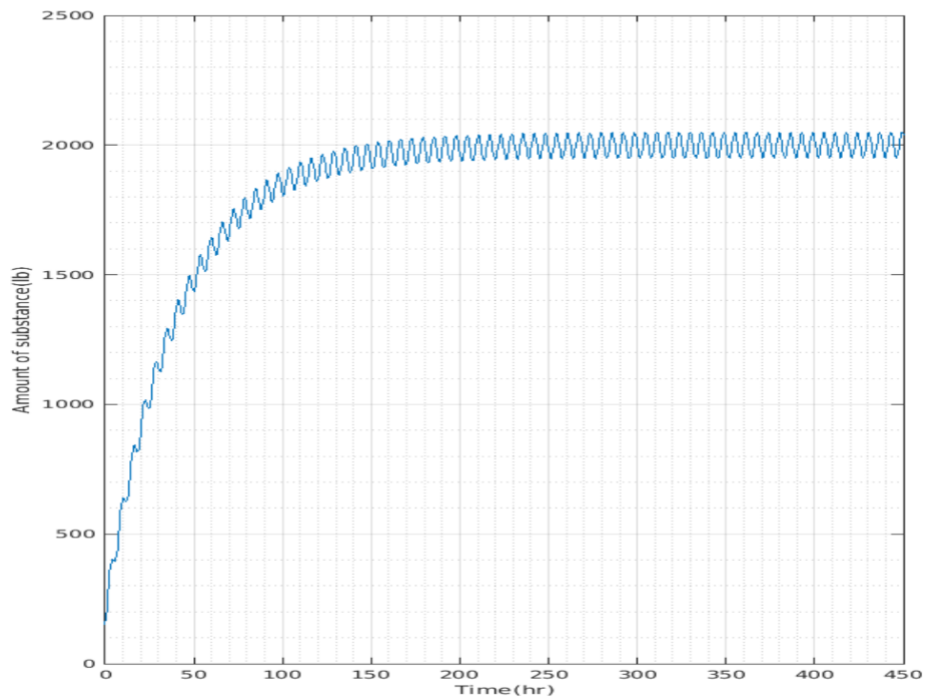
QUESTION 2:

```matlab
 1 commandwindow
 2 clearvars
 3 clc
 4 close all
 5 syms y1 t1
 6 y(t1)=(1000)+(50/1.0025)*sin(t1)+((50*0.05)/1.0025)*cos(t1)-802.49*exp(-0.05*t1)
 7 t1 = 1:2:499
 8 xlabel('Time values(min)')
 9 ylabel('Litre')
10 plot(t1,y(t1))
11 grid on
12 grid minor
```

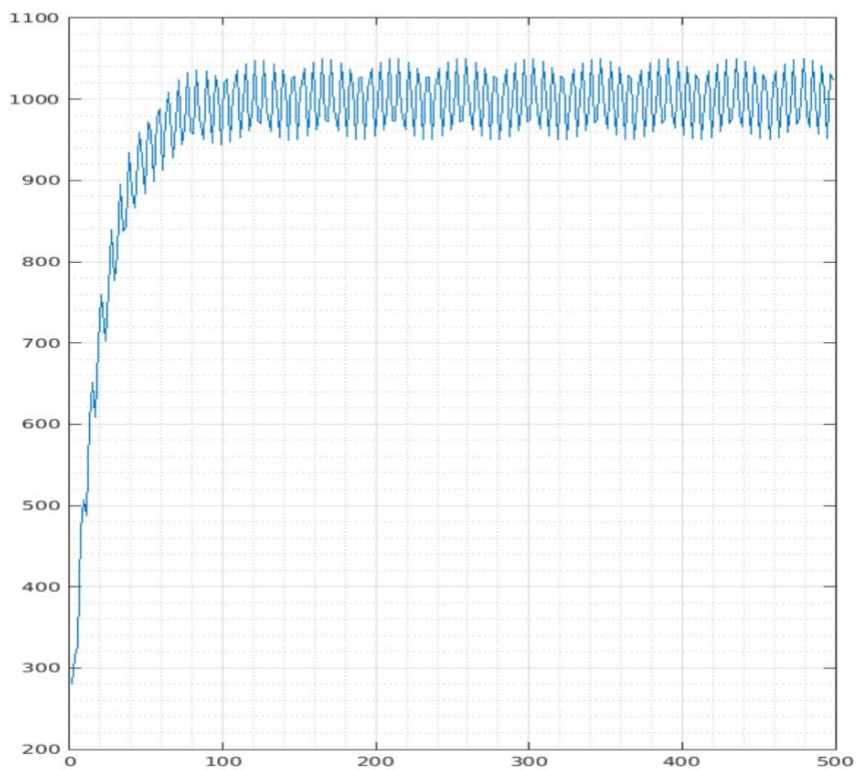← **Figures**   SHOW DATA CURSOR   ⋮

Figure 1: questiontwoa

```
1 commandwindow
2 clearvars
3 clc
4 close all
5
6 syms y2 t2
7 y(t2) = 1000-800*exp(-0.05*t2)
8 t2 = 0:2:500
9 xlabel('Time values(min)')
10 ylabel('Litre')
11 plot(t2,y(t2))
12 grid on
13 grid minor
```

← **Figures**   SHOW DATA CURSOR   ⋮

Figure 1: questiontwob
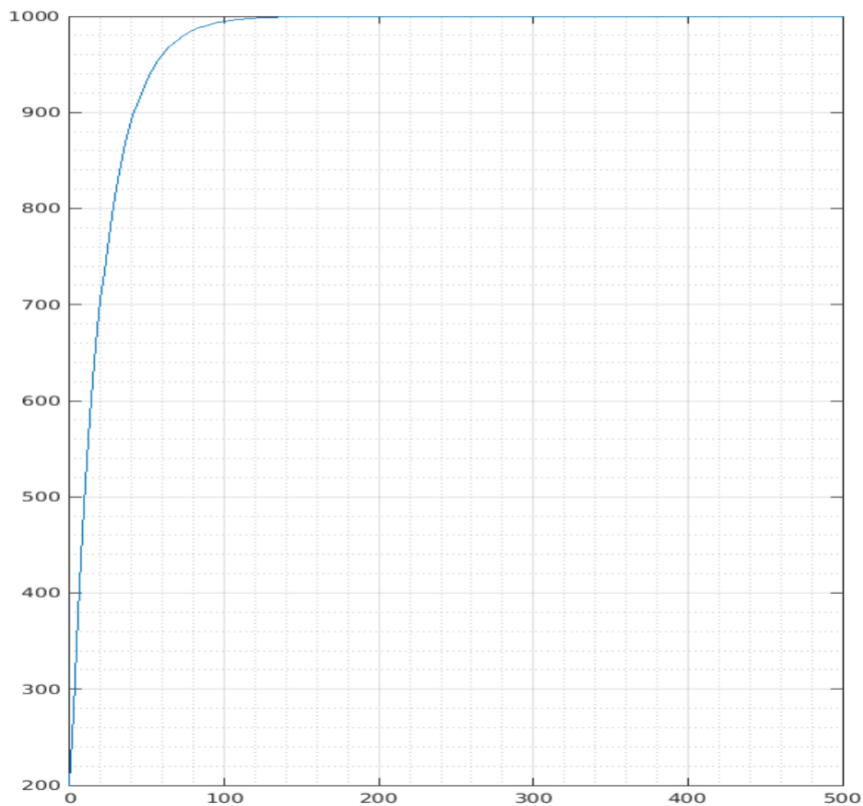
```
4 close all
5 syms Y1 T1 y1 y2 t1 t2
6 y(t1)=(1000)+(50/1.0025)*sin(t1)+((50*0.05)/1.0025)*cos(t1)-802.49*exp(-0.05*t1)
7 y(t2)=1000-800*exp(-0.05*t2)
8 t1=1:2:499
9 t2=0:2:500
10 T1 = [t1,t2]
11 Y1 = [y(t1),y(t2)]
12 xlabel('Time values(min)')
13 ylabel('Litre')
14 plot(T1,Y1)
15 grid on
16 grid minor
```

← **Figures** SHOW DATA CURSOR ⋮

Figure 1: questiontwocombine

QUESTION 2C:

```matlab
commandwindow
clear
clc
close all
syms Y T y1 y2 t1 t2
y(t1)=(1000)+(50/1.0025)*sin(t1)+((50*0.05)/1.0025)*cos(t1)-802.49*exp(-0.05*t1)
y(t2)=1000-800*exp(-0.05*t2)
t1=1:2:499
t2=0:2:500
T = [t1,t2]
Y = [y(t1),y(t2)]
xlabel('t(mins)')
ylabel('V(litres)')
plot(T,Y)
grid on
grid minor
Rows_Cols={'t(mins)','V(litre)'}
xlswrite('odevbesdata.xlsx',T(:),'veriler','A2')
xlswrite('odevbesdata.xlsx',Y(:),'veriler','B2')
xlswrite('odevbesdata.xlsx',Rows_Cols,'veriler','B1')
```

B2 : fx 279.963914100068

	A	B
1	t(min)	V(Litre)
2	1	279.9639
3	2	318.1907
4	3	313.8601
5	4	303.601
6	5	327.9009
7	6	393.9593
8	7	469.1423
9	8	511.0566
10	9	506.5922
11	10	484.0395
12	11	487.1398
13	12	534.9268
14	13	604.2824
15	14	651.2431
16	15	651.4694
17	16	622.6706
18	17	608.3676
19	18	637.9229
20	19	699.585
21	20	751.3315
22	21	759.541
23	22	729.9392
24	23	702.3679
25	24	714.1865
26	25	765.9535
27	26	820.9421
28	27	838.9333
29	28	813.2194
30	29	776.7953