ASalem Alabo Joshua
18/ENG06/007
Mechanical Engineering
ENG 282

Assignment

1a) Recall;

Accumulation rate of salt in the tank = Input rate of salt into tank − Output rate of salt from tank

$\therefore \frac{dm}{dt} = M_{in} - M_{out}$ ..... (*)

Where M = Amount of salt in tank

50 gal of brine run into tank per minute with each containing (1+sint) of dissolved salt.

$\therefore$ Input rate of salt (M_{in}) = 50(1+sint) lb/min

30 gal of brine run out of the tank containing that 1200 gal of water.

$\therefore$ Output rate of salt (M_{out}) = $\frac{30}{1200} \times m = 0.025m$ lb/min

Equation (*) becomes:

$\frac{dm}{dt}\left(\frac{lb}{min}\right) = 50(1+\sin t)\left(\frac{lb}{min}\right) - 0.025m\left(\frac{lb}{min}\right)$

$\therefore \frac{dm}{dt} = 50[(1+\sin t] - [0.025m]$ ..... (i)

Equation (i) is the ODE for solving the dynamics of amount of salt in tank.

1b) $\frac{dm}{dt} + 0.025m = 50(1+\sin t)$ ..... (ii)

Using integrating factor:

$\frac{dy}{dx} + Py = Q$ ..... (**)

Comparing equation (i) with (**)

let $\frac{dy}{dx} = \frac{dm}{dt}$, $P = 0.025$, $Q = 50(1+\sin t)$

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$I.F = e^{\int p\,dt}$ $\therefore I.F = e^{\int 0.025\,dt}$

$\therefore I.F = e^{0.025t}$

$M.IF = \int Q.IF\,dt$ ---(xxx)

$M.e^{0.025t} = \int 50(1+\sin t)\,e^{0.025t}\,dt$ ---(iii)

$Me^{0.025t} = 50\left[\int e^{0.025t}\,dt + \int \sin t\, e^{0.025t}\,dt\right]$ ---(v)

$Me^{0.025t} = 50\left[\frac{e^{0.025t}}{0.025} + \int \sin t\, e^{0.025t}\,dt\right]$ ---(v)

Solving $\int \sin t\, e^{0.025t}\,dt$ using Integration by parts

$\int e^{0.025t}\,dt = \int u\,dv\,dt$

$\int u\,dv\,dt = uv - \int v\,du\,dt$ ---(xxxx)

$u = e^{0.025t}$ $dv = \sin t$

$du = 0.025e^{0.025t}$ $v = -\cos t$

$\therefore \int e^{0.025t}\sin t\,dt = (e^{0.025t} \times -\cos t) - \int -\cos t \times 0.025e^{0.025t}\,dt$ (vi)

$= -e^{0.025t}\cos t + 0.025\int e^{0.025t}\cos t\,dt$ ---(vii)

$\int e^{0.025t}\cos t\,dt = \int u\,dv\,dt$; $u = e^{0.025t}$ $dv = \cos t$

$du = 0.025e^{0.025t}$ $v = \sin t$

$\therefore \int e^{0.025t}\cos t\,dt = e^{0.025t} \times \sin t - \int \sin t \times 0.025e^{0.025t}\,dt$ ---(viii)

$\therefore$ Equation (vii) becomes

$\int e^{0.025t}\sin t\,dt = -e^{0.025t}\cos t + 0.025\left[e^{0.025t} \times \sin t - \int \sin t \times 0.025e^{0.025t}\,dt\right]$ (ix)

$= -e^{0.025t}\cos t + 0.025\left[(e^{0.025t}\sin t) - 0.025\left[\int e^{0.025t}\sin t\,dt\right]\right]$ ---(x)

Let $I = \int e^{0.025t}\sin t\,dt$

$\therefore I = -e^{0.025t}\cos t + 0.025e^{0.025t}\sin t - 0.025^2 I$ ---(xi)

$I + 0.000625I = -e^{0.025t}\cos t + 0.025e^{0.025t}\sin t$ ---(xii)

$1.000625I = -e^{0.025t}(-\cos t + 0.025\sin t)$ ---(xiii)

$\int e^{0.025t}\sin t\,dt = I = \frac{e^{0.025t}}{1.000625}\left[-\cos t + 0.025\sin t\right]$ ---(xiv)

Substituting (xiv) into (v)

$me^{0.025t} = 50\left[\frac{e^{0.025t}}{0.025} + \frac{e^{0.025t}}{1.000625}(-\cos t + 0.025\sin t) + C\right]$ ---(xv)

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Dividing through by $e^{0.025t}$

$$m = \frac{50}{0.025} + \frac{50}{1.000625}(0.025\sin t - \cos t) + \frac{C}{e^{0.025t}} \quad \cdots (xvi)$$

$$m = 2000 + 49.9688(0.025\sin t - \cos t) + \frac{C}{e^{0.025t}} \quad \cdots (xv)$$

$$\therefore m = 2000 + 1.2492\sin t - 49.9688\cos t + \frac{C}{e^{0.025t}} \quad \cdots (xvi)$$

150lb of salt is dissolved initially

$\therefore$ at $t = 0$, $m = 150$

$\therefore$ equation (xvii) becomes

$$150 = 2000 + 1.2492\sin(0) - 9.9688\cos(0) + \frac{C}{e^{0.025(0)}} \quad \cdots (xvii)$$

$$150 = 2000 - 49.9688 + C \quad \cdots (xviii)$$

$$C = -1800.0312 \quad \cdots (xix)$$

Substituting (xix) into (xvi)

$$m = 2000 + 1.2492\sin t - 49.9688\cos t - \frac{1800.0312}{e^{0.025t}} \quad \cdots (xx)$$

$\therefore$ Equation (xx) gives the solution for the ODE for solving the dynamics of amount of salt in tank.

(c) $\Rightarrow t = 7.5\text{hrs} = 7.5 \times 60 = 450\text{mins}$.

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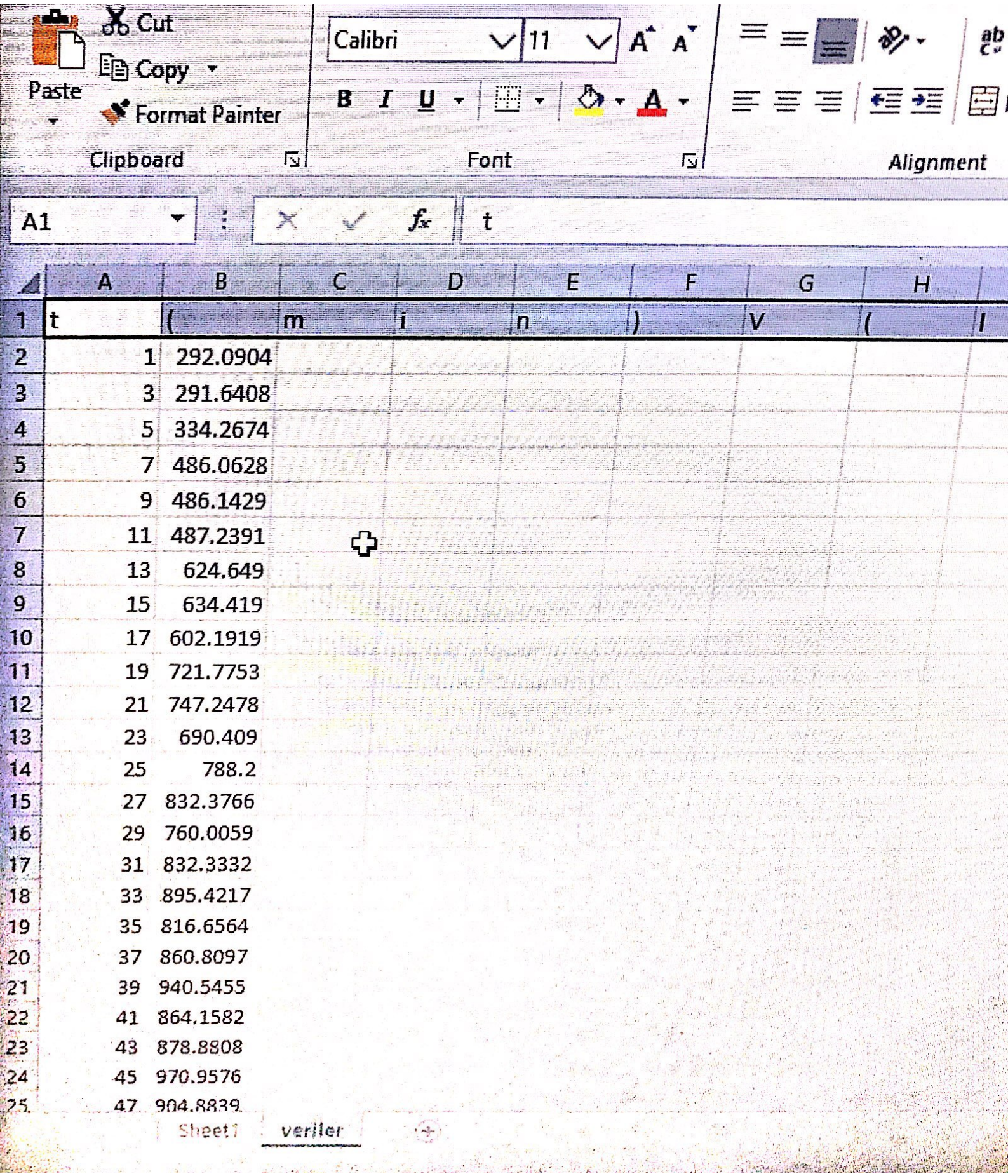

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Q1c.m | Q2.m | +

```
1 -    commandwindow
2 -    clear all
3 -    clc
4 -    syms m(t)
5 -    Y=diff(m)==50*(1+sin(t))-(0.025*m)
6 -    Cond=m(0)==150
7 -    Msol(t)=dsolve(Y,Cond)
8 -    t=0:0.5:450
9 -    plot (t,Msol(t))
10 -   grid on
11 -   grid minor
12 -   xlabel('Time,t (min)')
13 -   ylabel('Amount of salt in tank,m (lb)')
14
15
```

Command Window

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15
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Figure 1

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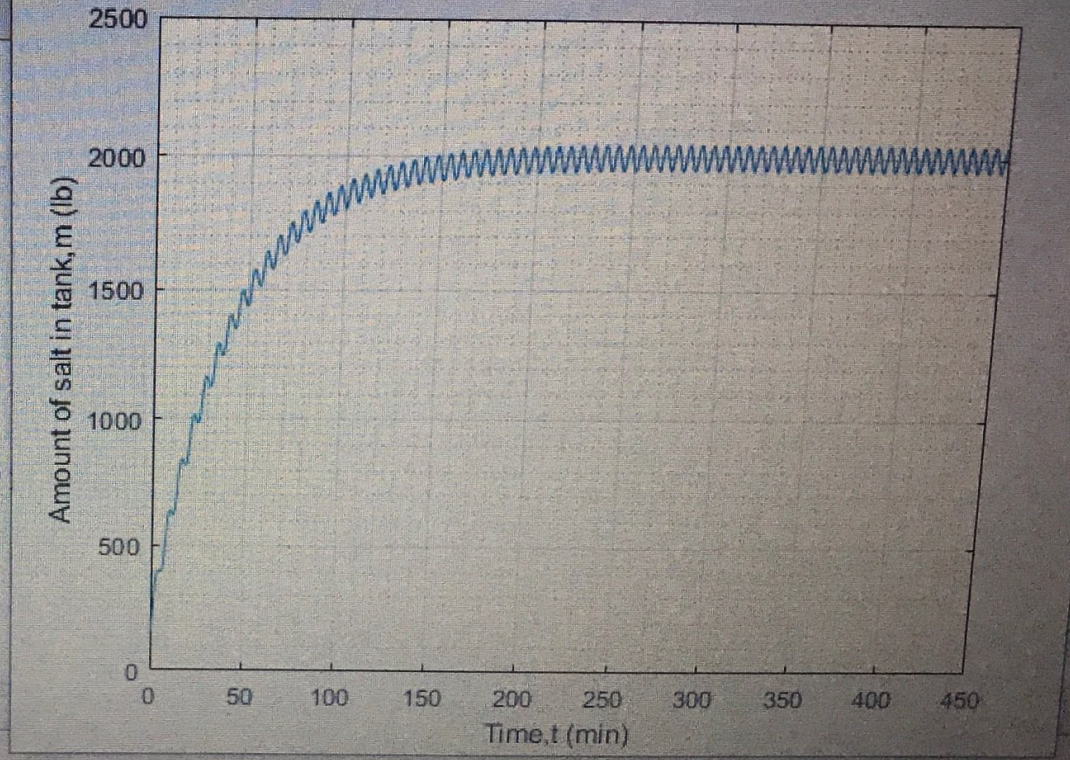

Command Window

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```
   444.0000   444.5000   445.0000   445.5000   446.0000   446.5000   447.0000   447.5000   448.0000   448.5000   449.0000   449.5000

  Column 901

   450.0000
```

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```
1 -  commandwindow
2 -  clear all
3 -  clc
4 -  t=1:2:499
5 -  tm=0:2:500
6 -  y=(50/0.05)+(50/1.0025)*sin(t)+(((50*0.5)/1.0025)*cos(t))-802.49*exp(-0.05*t)
7 -  ym=1000-800*exp(-0.05*tm)
8 -  time=[t,tm]
9 -  volume=[y,ym]
10 -  plot (time,volume)
11 -  xlabel('time,t(min)')
12 -  ylabel('volume, V(litre)')
13 -  col_header=['t(min)','V(litre)']
14 -  xlswrite('odevbesdata.xlsx',[time(:),volume(:)],'veriler','A2')
15 -  xlswrite('odevbesdata.xlsx',col_header,'veriler','A1')
```

Command Window

New to MATLAB? See resources for Getting Started.

```
  444.0000  444.5000  445.0000  445.5000  446.0000  446.5000  447.0000  447.5000  448.0000  448.5000

  Columns 901

  450.0000
```

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Editor - C:\Users\User\Documents\MATLAB\chika\MATLAB\Q2.m

Q1c.m | Q2.m | +

```
commandwindow
clear all
clc
t=1:2:499
tm=0:2:500
y=(50/0.05)+(50/1.0025)*sin(t)+(((50*0.5)/1.00
ym=1000-800*exp(-0.05*tm)
time=[t,tm]
volume=[y,ym]
plot (time,volume)
xlabel('time,t(min)')
ylabel('volume, V(litre)')
col_header=['t(min)','V(litre)']
xlswrite('odevbesdata.xlsx',[time(:),volume(:)
xlswrite('odevbesdata.xlsx',col_header,'verile
```

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lew to MATLAB? See resources for Getting Started.

```
    1.0000    1.0000    1.0000    1.0000    1.0000

col_header =

    't(min)V(litre)'
```

Figure 1

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