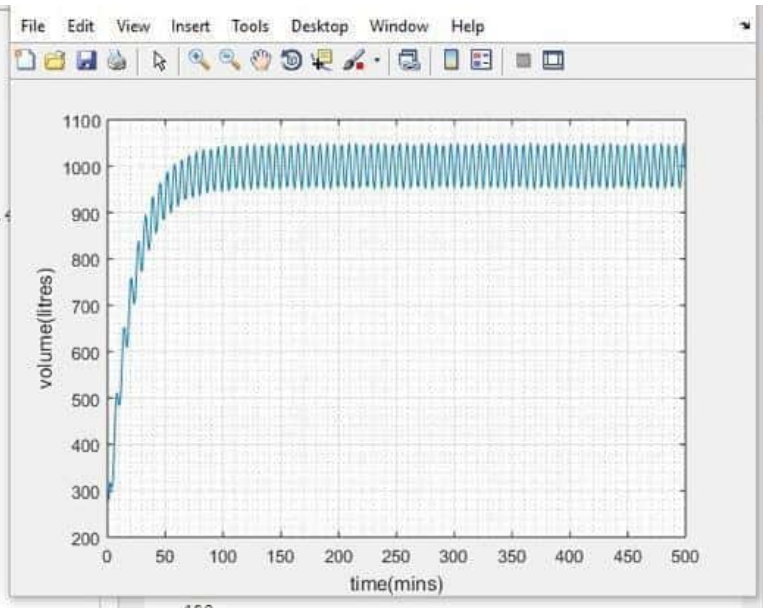
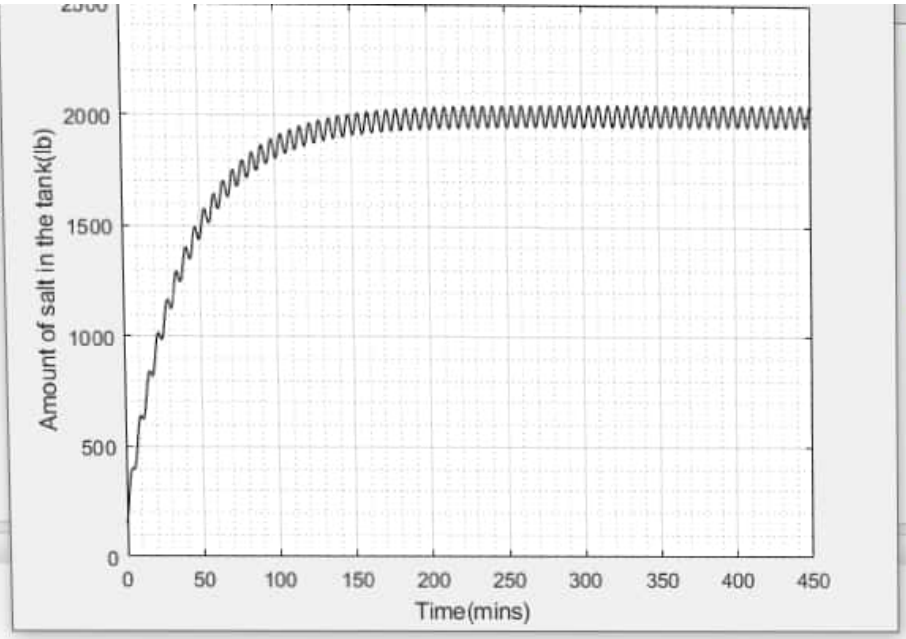
```
commandwindow
clear
clc
close all
syms t
values=[]
t=1:1:500
mean=1000-((exp(-0.05*t))*800)
y=1000+(50/1.0025)*sin(t)+(2.5/1.0025)*cos(t)-((exp(-0.05*t))*802.4

if rem(t,2) ==0
    values=[values,mean]
else
    values=[values,y]
end
excelvalues=transpose(values)
mins=transpose(t)
plot(t,values)
grid on
grid minor
xlabel ('time(mins)')
ylabel('volume(litres)')
xlswrite('odevbesdata.xlsx',{'t(min)'},'veriler','A1')
xlswrite('odevbesdata.xlsx',mins,'veriler','A2')
xlswrite('odevbesdata.xlsx',{'V(Litre)'},'veriler','B1')
xlswrite('odevbesdata.xlsx',excelvalues,'veriler','B2')
```




```
498
499
500
>>
```

```
commandwindow
clc
clear
close all
syms m(t) t equ
syms A(t)
equ = diff(m,t) == 50*(1 + sin(t)) - 0.025*m;
cond = m(0)==150;
A = dsolve(equ,cond)
t = [0:0.5:450]
An=subs(A,t)
plot(t,An,'black')
ylabel('Amount of salt in the tank(lb)')
xlabel('Time(mins)')
grid on
grid minor
```




```
mmand Window
  450.0000


An =

[ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, 2000 - (2000*1601^(1/2)*cos(atan(1
>>
```

1. Tank holds 1200 gal of water with 150 lb of salt dissolved in it.
- Input rate ⇒ 50 gal of brine/min [1 gal has (1+sin t) lb of salt]
- Output rate ⇒ 30 gal of brine/min

m ⇒ Amount of salt at anytime t

Using the balance law

$$\frac{dm}{dt} = m_{in} - m_{out}$$

1. 50 gal of water enter per minute and a gallon has (1+sin t) lb of salt

$$\therefore m_{in} = 50\,gal/min \times (1+\sin t) = 50(1+\sin t)\,lb/min$$

If the tank contains 1200 gal of water and 30 gal of water leaves per minute. Thats is $\frac{30}{1200} = \frac{1}{40} = 0.025$

⇒ 2.5% of water in the tank leaves per minute. This also means that 2.5% of salt will also leave the tank per minute

m_{out} ⇒ 2.5% of m

$$\frac{dm}{dt} = 50(1+\sin t) - 0.025m \quad \text{——— (i)}$$

b. differentiating using integration factor method

b. $$\frac{dm}{dt} + 0.025m = 50(\sin t + 1) \quad \text{——— (ii)}$$

$$\frac{dy}{dx} + Py = Q \left(\frac{dm}{dt} + Pm = Q\right)$$

$P = 0.025$, $Q = 50(\sin t + 1)$

$e^{\int P dt}$ = Integrating factor

$$\int P\,dt = \int 0.025\,dt = 0.025t$$

If ⇒ $e^{0.025t}$

$$40e^{u}\sin 40u + 1du = 40e^{u}(\sin 40u + 1) - 1600\frac{(40e^{u}\sin 40u + e^{u}\cos 40u)}{1601}$$

Put $U = t/40$

$$40e^{t/40}(\sin t + 1) - \frac{1600(40e^{t/40} + e^{t/40}\cos(t))}{1601}$$

$$\int 50e^{t/40}(\sin t + 1)dt$$

$$\int 50e^{t/40}(\sin t + 1)dt \Rightarrow \frac{2000e^{t/40}(\sin t - 40\cos t + 1601)}{1601} + C \quad -(V)$$

Recall the original integration factor

$$m.IF = \int Q.IF\,dt$$

$$m\cdot e^{0.025t} = \int 150(\sin t + 1)\cdot e^{0.025t}\,dt$$

put eqn (V)

$$m\cdot e^{0.025t} = 2000e^{0.025}\frac{(\sin t - 40\cos t + 1601)}{1601} + C$$

$$m = \frac{2000(\sin t - 40\cos t + 1601)}{1601} + m_0\cdot e^{-0.025t} \quad -(VI)$$

at $t = 0$ min and $m = 150$ lb of salt

$$150 = \frac{2000(\sin 0 - 40\cos(0) + 1601)}{1601} + m_0\cdot 1$$

$$150 = 1950.03 + m_0$$

$$m_0 = -1800.03$$

put m_0 in eqn (VI)

$$m = \frac{2000(\sin t - 40\cos t + 1601)}{1601} - \frac{1800.03}{e^{0.025t}}$$

From $y \cdot IF = \int Q \cdot IF\, dx$

$m \cdot IF = \int Q \cdot IF\, dt$

$m \cdot e^{0.025t} = \int 50(\sin t + 1) \cdot e^{0.025t}\, dt$

$= 50 \int e^{t/40} (\sin t + 1) dt$

let $v = \frac{t}{40} \rightarrow \frac{dv}{dt} = \frac{1}{40}$, $dt = 40\, dv$

$= 40 \int e^v (\sin 40v + 1) dv$

$\int e^v (\sin 40v + 1) dv$

Let $q = \sin 40v + 1$, $dp = e^v dv$

$\frac{dq}{dv} = 40 \cos 40v$ $\int dp = \int e^v dv$

$p = e^v$

$dq = 40(\cos 40v) dv$

From $\int q\,dp = qp - \int p\,dq$

$\int 40e^v (\cos 40v) dv \Rightarrow 40 \int e^v (\cos 40v) dv$

$\int e^v \cos 40v\, dv \Rightarrow$ Let $q = \cos 40v$ $dp = e^v dv$

Integrating

$\int e^v \cos 40v\, dv = e^v \cos 40v - \int -40e^v (\sin 40v) dv$ — (iii)

integration by part

$q = -40 \sin 40v$ $dp = e^v$

$dq = -1600(\cos 40v) dv$ $p = e^v$

$\int e^v \cos 40v\, dv = e^v \cos 40v - C - 40e^v \sin 40v - \int -1600 e^v \cos 40v\, dv$

$1601 \int e^v \cos 40v\, dv = e^v \cos 40 + 40e^v \sin 40v$

$\int e^v \cos 40v\, dv = \frac{e^v \cos 40 + 40e^v \sin 40v}{1601}$

$40 \int e^v \cos 40v\, dv = 40(40e^v \sin 40v + e^v \cos 40v)$ — (iv)

From (iii)

$\Rightarrow \int e^v (\sin 40v + 1) dv = e^v \sin 40v + 1 - \int 40e^v \cos 40v\, dv$

$\int e^v (\sin 40v + 1) \Rightarrow e^v \sin 40v + 1 - \frac{40(40e^v \sin 40v + e^v \cos 40v)}{1601}$