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Matric: 19/ENG05/036
MAT 102
Assignment

1. If $M = Pi - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$, find the value of P which

a. M and N are perpendicular to each other

Solution

$\bar{M} \cdot \bar{N} = (Pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4P - 18 + 3$

For perpendicular vectors $\bar{M} \cdot \bar{N} = 0$

$0 = 4P - 15$

$4P = 15$

$P = \frac{15}{4}$

b. M, N, and O are Coplanar

Solution

$\bar{M} \cdot (\bar{N} \times \bar{O}) = 0$: For coplanar Vectors

$$\bar{M} \cdot (\bar{N} \times \bar{O}) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= P\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$= P(6-3) + 6(8+1) - 3(-12-3)$

$= 3P + 54 + 45$

$0 = 3P + 99$

$-99 = 3P$

$P = -33$

2. Find the direction cosines and the unit vectors along the sum of $3i + 2j + 5k$, $2i - j + 6k$ and $5i + 2j - 3k$

Solution

$(3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{A} = 10i + 3j + 8k$

Direction Cosines of $\bar{A}$

~~Cos~~ α

$|\bar{A}| = \sqrt{10^2 + 3^2 + 8^2}$

$= \sqrt{173}$

$\cos\alpha = \frac{a_x}{|\bar{A}|} = \frac{10}{\sqrt{173}} = 0.7603$

$\cos\beta = \frac{a_y}{|\bar{A}|} = \frac{3}{\sqrt{173}} = 0.2281$

$\cos\gamma = \frac{a_z}{|\bar{A}|} = \frac{8}{\sqrt{173}} = 0.6082$

Unit vector $= \dfrac{\bar{A}}{|\bar{A}|}$

$= \dfrac{10i + 3j + 8k}{\sqrt{173}}$

3. If $F = 3ui + u^2 j + (u+2)k$ and $V = 2ui - 3uj + (u-2)k$, evaluate the integral of $(F \times V)\,du$ from 0 to 1

Solution

$$F \times V = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$\cancel{= i(u^3 - 2u^2 + 3u^2 - 6u) - j(3u^2 - 6u - 2u^2 + 4u)}$$

$$= i((u^3 - 2u^2) - (-3u^2 - 6u)) - j((3u^2 - 6u) - (2u^2 + 4u)) + k(-9u^2 - 2u^3)$$

$$\int_0^1 (F \times V)\,du = \int_0^1 \left[(u^3 - 2u^2 + 3u^2 + 6u)i - (3u^2 - 6u - 2u^2 - 4u)j + (-9u^2 - 2u^3)k\right] du$$

$$\int_0^1 (F \times V)\,du = \int_0^1 \left[(u^3 - u^2 + 6u)i - (u^2 - 10u)j + (-9u^2 - 2u^3)k\right] du$$

$$= \left[\frac{u^4}{4} - \frac{u^3}{3} + 3u^2\right]i - \left[\frac{u^3}{3} - 5u^2\right]j + \left[-3u^3 - \frac{u^4}{2}\right]k$$

$$= \left[\frac{(1^4 - 0^4)}{4} - \frac{(1^3 - 0^3)}{3} + 3(1^2 - 0^2)\right]i - \left[\frac{(1^3 - 0^3)}{3} - 5(1^2 - 0^2)\right]j + \left[-3(1^3 - 0^3) - \frac{(1^4 - 0^4)}{2}\right]k$$

$$= \left[\frac{1}{4} - \frac{1}{3} + 3\right]i - \left[\frac{1}{3} - 5\right]j + \left[-3 - \frac{1}{2}\right]k$$

$$= \frac{35}{12}i + \frac{14}{3}j - \frac{7}{2}k$$