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18/ENG 02/065

Computer Engineering

1) Accumulation rate of salt with a system = input − output

$$\frac{dy}{dx} = y_{in} - y_{out}$$

Since 50 gal enter per minute and the gal contains $(1+\sin t)$ lb ie the amount of salt entering tank is

$$y_{in} = \frac{50\,gal}{min} \times (1+\sin t)\frac{lb}{gal} = 50(1+\sin t)\frac{lb}{min}$$

∴ Since the tank contains 120 gal of water and runs out at 30 gal per min

$$\frac{30\,gal}{1200\,gal} = \frac{1}{40} = 0.025 \text{ ie } 25\% \text{ salt per sint}$$

in the tank also leaves the tank per/min ie y_{out} $0.025y$ or 2.5% of y

Therefore from formular (equ(1))

$$\frac{dy}{dt} = 50(1+\sin t)\,lb/min - 2.5\% \text{ of } y\,lb/min$$

$$\frac{dy}{dt} = 50(1+\sin t) - 0.025y$$

$$\frac{dy}{dt} = -0.025y + 50(1+\sin t)$$

$$\frac{dy}{dt} + 0.025y = 50(1+\sin t)$$

using linear equation

$$\frac{dy}{dx} + Py = Q$$

$$P = 0.025,\ Q = 50(1+\sin t)$$

$$\int P\,dt = 0.025t \quad if = e^{\int P\,dt} = e^{0.025t}$$

$$y * if = \int Q * if \cdot dt$$

$$y e^{0.025t} = \int 50(1+\sin t)e^{0.025t}dt$$

$$50\int e^{0.025t} + e^{0.025}\sin t\,dt$$

$$y e^{0.025t} = \int 50(1+\sin t)e^{0.025}dt$$

$$= 50\int e^{0.025t} + e^{0.025dt}$$

$$\frac{50 \cdot e^{0.025}}{0.025} + \int e^{0.025t}\sin t \cdot dt$$

using integration by partial fraction

$$\int e^{0.025t}\sin t\,dt$$

$$u = e^{0.025t} \quad dv = \sin t$$

$$du = 0.025t \quad dv = \sin t$$

$$\int e^{0.025t}\sin t = e^{0.025t} \cdot -\cos t - \int -\cos t$$

$$= e^{0.025t}\cos t - \int \cos t \cdot 0.025e^{0.025t} +$$

$$e^{0.025t}\cos t + 0.025\int e^{0.025} + c$$

using integration by partial fraction

$$\int u\,dv = uv - \int v\,du$$

$$u = e^{0.025t}, \quad dv = \cos t$$

$$du = 0.025e^{0.025t}$$

$$= -e^{0.025t}\cos t + 0.025\left[e^{0.025t} \cdot \sin t - \int \sin t + c - 0.025e^{0.025t}\right]$$

~~$e^{0.025t}\cos t + 0.025$~~

Let $Q = \int e^{0.025t}$

$$Q = e^{0.025}\cos t + 0.025\left[e^{0.025t}\sin t - 0.025Q\right]$$

$$Q = e^{0.025}\cos t + 0.025e^{0.025} - 6.25^{-4}Q$$

$$Q + 6.25^{-4}Q = e^{0.025t}\cos t + 0.025e^{0.025t}$$

$Q + 6.25^{-4} Q = -e^{0.025t} \cos t + 0.025 e^{0.025t}$

$Q + 0.000625 Q = e^{0.025t} \cos t + 0.025 e^{0.025t}$

$1.000625 Q = -e^{0.025t} \cos t + 0.025 e^{0.025t}$

$1.000625 Q = -e^{0.025t} \cos t + 0.025 e^{0.025t}$

$Q = \frac{-e}{1.000625} (\cos t - 0.025) + c$

Recall $\int e^{0.025t} \sin t = \frac{-e^{0.025t}}{1.000625} (\cos t - 0.025) + c$

Since $\int e^{0.025t} \sin t = \frac{-e^{0.025t}}{1.000625} (\cos t - 0.025) + c$

$y e^{0.025t} = 2000 e^{0.025t} - \frac{50 \cdot e^{0.025t}}{1.000625} (\cos t - 0.025) + 50c$

divide through by $e^{0.025t}$

$y = 2000 - \frac{50}{1.000625} (\cos t - 0.025) + \frac{50c}{e^{0.025t}}$

$\therefore$ when $y = 150$ lb and $t = 0$ min

$150 = 2000 - \frac{50}{1.000625} (1 - 0) + \frac{50c}{1}$

$150 = 2000 - 49.968 (1) + 50c$

$150 = 1950.032 + 50c$

$-\frac{1800.032}{50} = \frac{50c}{50}$

$c = -36.00064$ (amount of salt in tank at anytime

```
commandwindow
clear
clc
close all
syms m t
ans=dsolve('Dm+0.025*m=50+50*sin(t)','m(0)=150')
t=0:0.5:450
tn=subs(ans,t)
plot(t,tn)
```

Command Window

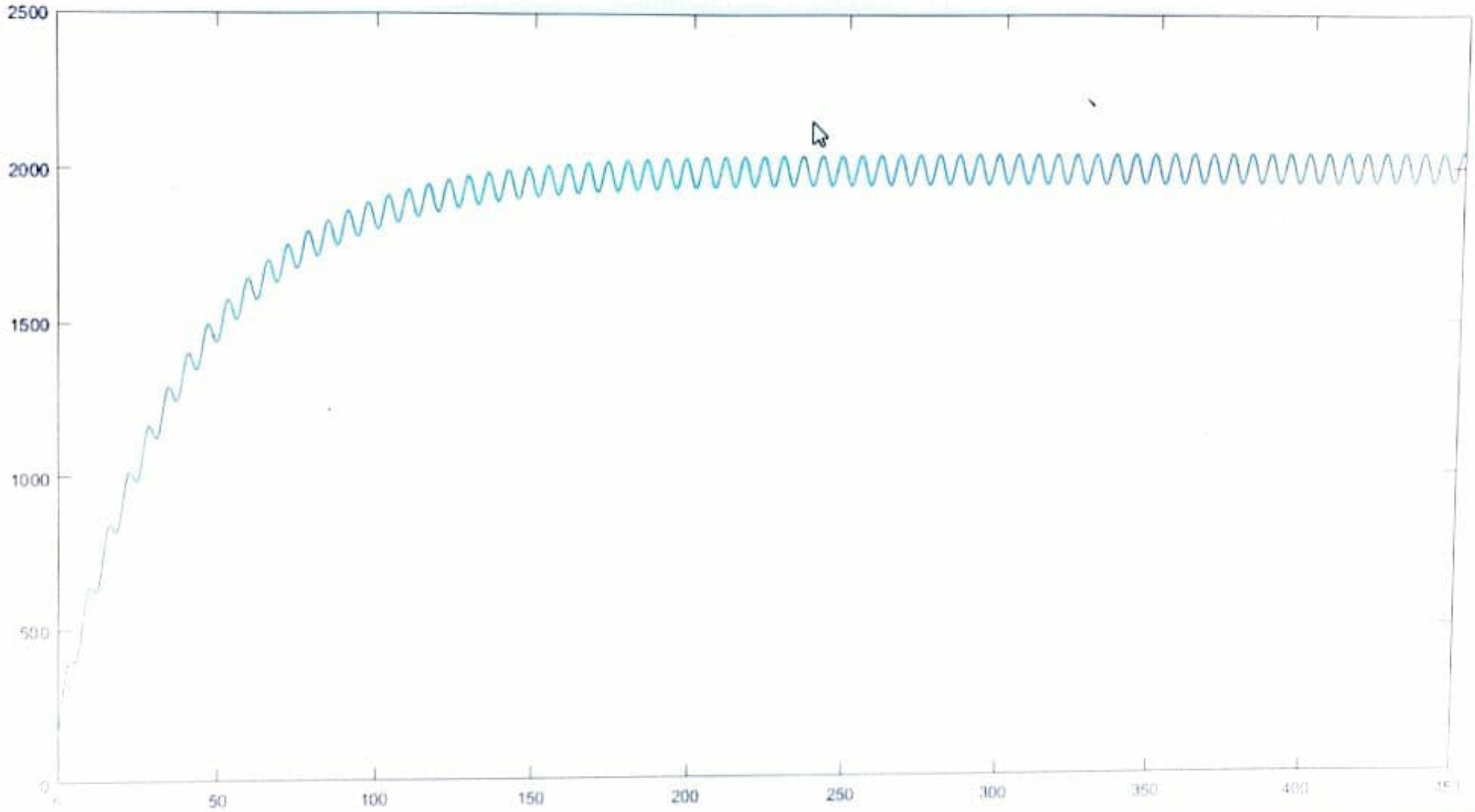
New to MATLAB? See resources for Getting Started.

```
    445.5000   446.0000   446.5000   447.0000   447.5000   448.0000   448.5000   449.0000   449.5000   450.0

  tn =

  [ 150, 2000 - (2000*1601^(1/2)*cos(atan(1/40) + 1/2))/1601 - (2881850*exp(-1/80))/1601, 2000 - (20
>>
```

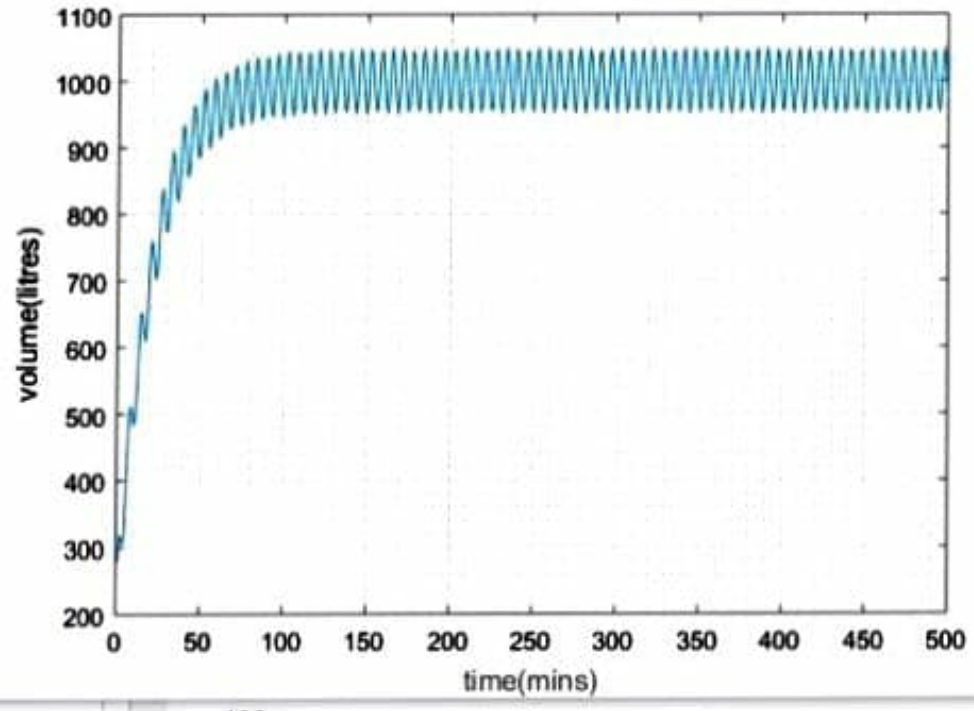
Insert Tools Desktop Window Help



```
commandwindow
clear
clc
close all
syms t
values=[]
t=1:1:500
mean=1000-((exp(-0.05*t))*800)
y=1000+(50/1.0025)*sin(t)+(2.5/1.0025)*cos(t)-((exp(-0.05*t))*802.4

if rem(t,2) ==0
    values=[values,mean]
else
    values=[values,y]
end
excelvalues=transpose(values)
mins=transpose(t)
plot(t,values)
grid on
grid minor
xlabel ('time(mins)')
ylabel('volume(litres)')
xlswrite('odevbesdata.xlsx',{'t(min)'},'veriler','A1')
xlswrite('odevbesdata.xlsx',mins,'veriler','A2')
xlswrite('odevbesdata.xlsx',{'V(Litre)'},'veriler','B1')
xlswrite('odevbesdata.xlsx',excelvalues,'veriler','B2')
```

File Edit View Insert Tools Desktop Window Help




```
498
499
500
```

fx >>