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Computer Science

1) If $M = Pi - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$, find the value of P for which

a) M and N are perpendicular to each other

solution

$M \cdot N = (Pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4P - 18 + 3$

$= 4P - 15$

for perpendicular vectors

$4P - 15 = 0$

$4P = 15$

$P = 15/4$

b) M, N and O are coplanar

$$M \cdot (N \times O) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$P\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$P(6 - 3) + 6(8 + 1) - 3(-12 - 3)$

$3P + 54 + 45 = 0$

$3P + 99 = = 0$

$3P = -99$

$P = -99/3$

$= -33$

2 Find the direction cosines and the unit vector along the sum of $3i + 2j + 5k$, $2i - j + 6k$ and $5i + 2j - 3k$

solution

$\bar{V} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{V} = 10i + 3j + 8k$

$a_x = 10 \quad a_y = 3 \quad a_z = 8$

$|V| = \sqrt{10^2 + 3^2 + 8^2}$

$= 13.15$

The direction cosines are

$\cos\alpha = \frac{a_x}{|V|} = \frac{10}{13.15} = 0.760$

$\cos\beta = \frac{a_y}{|V|} = \frac{3}{13.15} = 0.228$

$\cos\gamma = \frac{a_z}{|V|} = \frac{8}{13.15} = 0.608$

Unit vector $e_v = \frac{V}{|V|} = \frac{10i + 3j + 8k}{13.15}$

$= \frac{10i}{13.15} + \frac{3}{13.15}j + \frac{8}{13.15}k$

3) If $F = 3ui + u^2 j + (u+2)k$ and $V = 2ui - 3uj + (u-2)k$, evaluate the integral of $(F \times V)\,du$ from 0 to 1.

Solution

$$F \times V = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$i(u^3 - 2u + 3u^2 + 6u) - j(3u^2 - 6u - 2u^2 - 4u) + k(-9u^2 - 2u^3)$

$i(u^3 + 3u^2 + 4u) - j(u^2 - 10u) + k(-9u^2 - 2u^3)$

$$\int_0^1 (F \times V)\,du = \int_0^1 \left[(u^3 + 3u^2 + 4u)i - (u^2 - 10u)j + (-9u^2 - 2u^3)k\right] du$$

$$= i\left(\frac{u^4}{4} + \frac{3u^3}{3} + \frac{4u^2}{2}\right)\Bigg|_0^1 - j\left(\frac{u^3}{3} - \frac{10u^2}{2}\right)\Bigg|_0^1 + k\left(-\frac{9u^3}{3} - \frac{2u^4}{4}\right)\Bigg|_0^1$$

$$= i\left(\frac{u^4}{4} + u^3 + 2u^2\right]_0^1 - j\left(\frac{u^3}{3} - 5u^2\right|_0^1 + k\left(-3u^3 - \frac{u^4}{2}\right|_0^1$$

$$= i\left[\left(\frac{1^4}{4} + 1^3 + 2(1)^2\right) - \left(\frac{0^4}{4} + 0^3 + 2(0)^2\right)\right] - j\left[\left(\frac{1^3}{3} - 5(1)^2\right) - \left(\frac{0^3}{3} - 5(0)^2\right)\right]$$

$$+ k\left[\left(-3(1)^3 - \frac{1^4}{2}\right) - \left(-3(0)^3 - \frac{0^4}{2}\right)\right]$$

$$= i\left[\frac{1}{4} + 1 + 2 - (0)\right] - j\left[\frac{1}{3} - 5 - (0)\right] + k\left[-3 - \frac{1}{2} - (0)\right]$$

$$= i\left[\frac{13}{4}\right] - j\left[-\frac{14}{3}\right] + k\left[-\frac{7}{2}\right]$$

$$= \frac{13}{4}i + \frac{14}{3}j - \frac{7}{2}k$$