

MAT 104 ASSSIGNMENT

$$\frac{1}{(x-1)(x-2)(x-3)}$$

7. First

Partial fraction decomposition

$$\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

Multiplying both sides $(x-1)(x-2)(x-3)$

$$1 = (x-2)(x-3)A + B(x-1)(x-3) + C(x-1)(x-2)$$

$$1 = Ax^2 - 5A + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2Cx$$

$$1 = (A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C)$$

$$1 = (A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C)$$

$$-1 = 6A + 3B + 2C \quad \dots (1)$$

$$0 = -5A - 4B - 3C \quad \dots (2)$$

$$0 = A + B + C \quad \dots (3)$$

Sub (3) in (1)

$$A = -B - C$$

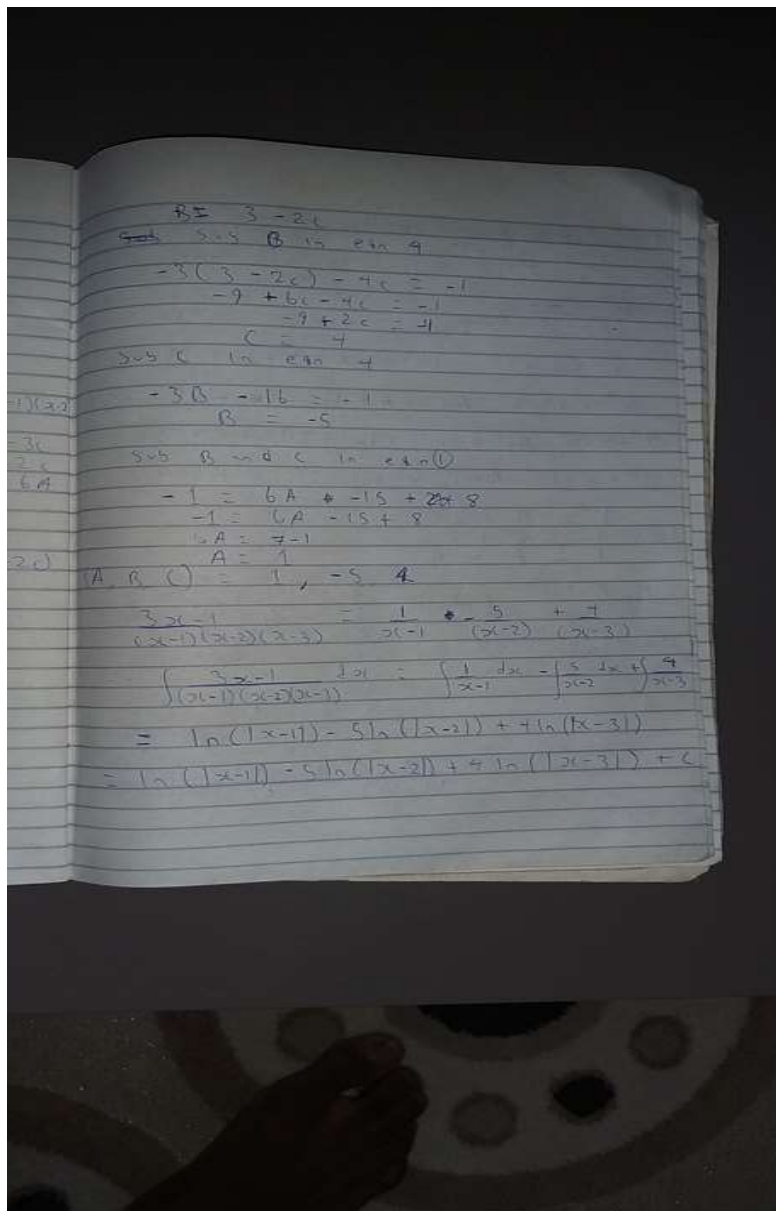
Sub in eqn (1) and (2)

$$6(-B-C) + 3B + 2C = -1$$

$$-5(-B-C) - 4B - 3C = 3$$

$$-3B - 4C = -1 \quad \dots (4)$$

$$B + 2C = 3 \quad \dots (5)$$



$$2. \frac{x^2 + 2x + 1}{(x+2)(x^2+1)}$$

Partial fraction decomposition

$$\frac{x^2 + 2x + 1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$x^2 + 2x + 1 = (x+2)(A) + (Bx+C)(x^2+1)$$

$$x^2 + 2x + 1 = Ax^2 + 2Ax + Ax^2 + Bx^3 + Cx^2 + Bx + C$$

$$x^2 + 2x + 1 = Ax^2 + Bx^3 + (2A + C)x^2 + (2B + B)x + (A + C)$$

$$x^2 + 2x + 1 = (B + A)x^3 + (2A + C)x^2 + (2B + B)x + (A + C)$$

$$1 = A + 2C$$

$$1 = 2A + B$$

$$1 = A + C$$

$$(A \ B \ C) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\therefore \frac{x^2 + 2x + 1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$= \frac{3}{x+2} + \frac{2x+1}{x^2+1}$$

$$= \frac{3}{x+2} + \frac{2x+1}{x^2+1}$$

$$= \int \frac{1}{5(x+2)} dx + \int \frac{2x+1}{5(x^2-1)} dx$$

$$= \frac{1}{5} \ln|x+2| + \frac{1}{5} \ln|x^2-1| + \frac{2x+1}{5}$$

$$= \frac{1}{5} \ln|2x+2| + \frac{1}{5} \ln|x^2-1| + \frac{2x+1}{5}$$

c)
2c

$$\int \frac{x^2+1}{(x-3)(x-2)^2} dx$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2+1 = (x-2)^2 \cdot A + (x-3) \cdot (x-2) \cdot B + (x-3) \cdot C$$

$$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$$

$$= (A+B)x^2 + (-4A-5B+C)x + (4A+6B-3C)$$

$$x^2+1 = (A+B)x^2 + (-4A-5B+C)x + (4A+6B-3C)$$

$$\begin{cases} 1 = 4A + 6B - 3C \\ 0 = -4A - 5B + C \\ 1 = A + B \end{cases}$$

$$(A \ B \ C) = (10 \ -2 \ -4)$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{10}{x-3} + \frac{-2}{x-2} + \frac{-4}{(x-2)^2}$$

$$\therefore \int \frac{10 \ln x - 9}{x-2} dx = \int \frac{9 \ln x}{x-2} dx + \int \frac{5}{(x-2)^2} dx$$

$$10 \ln(|x-2|) - 9 \ln(|x-2|) + \frac{5}{x-2}$$

$$= 10 \ln(|x-2|) - 9 \ln(|x-2|) + \frac{5}{x-2} + C$$

4) $\int \frac{x^3 + 2x^2 + 2x + 1}{x-1} dx$

Separate like fractions

$$\left[\frac{x^3}{x-1} + \frac{2x^2}{x-1} + \frac{2x}{x-1} + \frac{1}{x-1} \right] dx$$

$$\int \frac{x^3}{x-1} dx + \int \frac{2x^2}{x-1} dx + \int \frac{2x+1}{x-1} dx$$

~~$$2x^3 + \frac{5x^2}{6} + 3x - 1 + \ln(|x-1|) + \frac{2x^2}{2} + \frac{1}{2}$$~~

$$2x^3 + \frac{5x^2}{6} + 3x - 1 + \ln(|x-1|) + 2x^2 + x + \frac{1}{2}(x-1)$$

$$+ \frac{1}{2}(x-1) + \ln(|x-1|) = \ln(|x-1|) + C$$

$$= 2x^3 + \frac{5x^2}{6} + 3x - 1 + 4 \ln(|x-1|) + C$$